STRUCTURAL MECHANICS.

STRUCTURAL MECHANICS.

A HANDBOOK FOR ENGINEERS, ARCHITECTS, AND STUDENTS.

BY

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PREFACE.

THE following pages have been written with the view of supplying a want which the author has himself experienced in the study of practical mechanics; for, although there are many treatises on the subject, there is not one which takes the student direct from the simpler to the more complicated problems which occur in practice. The elementary portion is, however, treated in as short a way as possible, the chief part of the space being devoted to the solution of problems which can only be gathered from a number of different books, such as the works of Baker, Barlow, Box, and Twisden, and the papers by Bell, Christie, Clarke, and Fidler, in the Proceedings of the Institution of Civil Engineers and American Society of Civil Engineers, and these the author has freely consulted.

While designed for the student, it is intended that the work should be useful for every-day reference in the engineer's and architect's office, and for this purpose the examples which are given are all either of practical application or else are designed for the purpose of comparing calculated results with actual experiments, and so establishing the formulæ in the most satisfactory way.

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STRUCTURAL MECHANICS.

CHAPTER I.

DATA.—WEIGHTS OF MATERIALS.—EFFECT OF HEAT.—RESISTANCE TO TENSION AND COMPRESSION.—FACTORS OF SAFETY.—LOADS ON ROOFS, FLOORS, AND BRIDGES.—SUPPORTING POWER OF FOUNDATIONS.

1. It is assumed in the following treatise that the reader is acquainted with the elements of Algebra, Euclid, Trigonometry, and Mechanics; but a knowledge of the higher branches of mathematics is not required.

As regards Algebra, the use of quadratic equations is occasionally necessary, and the following summations are sometimes made use of:—

If $s = a + (a + b) + (a + 2b) + \ldots + l$,

$$\text{Then } s = \frac{n}{2}(a + l) = \frac{n}{2}\left\{2a + (n-1)b\right\}.$$

If $s = 1^2 + 2^2 + 3^2 + \ldots + n^2$, Then $s = \dfrac{n(n+1)(2n+1)}{6}$.

If $s = 1^3 + 2^3 + 3^3 + \ldots + n^3$, Then $s = \left\{\dfrac{n(n+1)}{2}\right\}^2$.

If $s = \sin\dfrac{\pi}{n} + \sin\dfrac{2\pi}{n} + \sin\dfrac{3\pi}{n} + \ldots + \sin\dfrac{n\pi}{n}$,

$$\text{Then } s=\frac{\sin\left(\frac{\pi}{n}+\frac{n-1}{2}\frac{\pi}{n}\right)\sin\frac{n\pi}{2n}}{\sin\frac{\pi}{2n}},$$

$$=\frac{\sin\left(\frac{\pi}{2n}+\frac{\pi}{2}\right)}{\sin\frac{\pi}{2n}},$$

$$=\frac{\sin\frac{\pi}{2n}\cos\frac{\pi}{2}+\cos\frac{\pi}{2n}\sin\frac{\pi}{2}}{\sin\frac{\pi}{2n}},$$

$$=\cot\frac{\pi}{2n},$$

$$=\frac{2n}{\pi}.$$

for cot $\frac{\pi}{2n}$ may be written $\dfrac{n}{n\tan\frac{\frac{\pi}{2}}{n}}$ which equals $\dfrac{n}{\frac{\pi}{2}}$, since the limit of $n\tan\dfrac{\frac{\pi}{2}}{n}$, when n is indefinitely increased is $\frac{\pi}{2}$.

$$\text{Also } (1+x)^n=1+nx+\frac{n(n-1)}{1.2}x^2+\frac{n(n-1)(n-2)}{1.2.3}x^3+\ldots$$

In Mechanics, the parallelogram and triangle of forces, the principles of moments and couples and the properties of the centre of gravity, are assumed to be understood, as also are the fundamental laws of the pressure of water.

2. The following may be taken as the weights of some of the principal materials used in construction or affecting structures, in pounds :—

Water, Fresh	$62\frac{1}{2}$	per cub. foot,	or 10	per gallon.
„ Salt	64	„	„ $10\frac{1}{4}$	„
Oil	56	„	„ 9	„
Wrought Iron	480	„	„ 10*	per. sq. foot $\frac{1}{4}$ thick.
Cast Iron	456	„	„ 10†	„ „
Steel	492	„	„ $10\frac{1}{4}$*	„ „
Fir	37	„		
Yellow Pine	28	„		
Pitch Pine	37	„		
Teak	56	„		
Oak, English	46	„		
„ Bog	66	„		
Ash	47	„		
Beech	43	„		
Elm	34	„		
Slate	180	„		
Granite	170	„		
Flint	164	„		
Sandstone	140	„		
Masonry	140	„		
Brickwork	112	„		

3. A rise of temperature of 1° Cent. will lengthen a bar of iron ·000011 of its length, and so a variation of 45°, which is the maximum we need in general allow for, will alter the length ·0005; and if the bar is fixed at both ends, this, as will be seen in what follows (Chapter iv.), will induce a strain of 5 tons per square inch in the metal.

4. The following table gives what we will consider as the resistances to tension and compression of some of the most commonly used materials. These resistances are very variable, but as a large Factor of safety

* It is usual to add 5 per cent. for rivets and bolts.

† Allowance being made for inaccurate casting.

With most of these materials the actual weights vary from some 5 per cent. over to 5 per cent. under the weights given.

is always taken, they are sufficient for our purpose. They are given in tons per square inch.

	Tension.			Compression.		
	Ultimate.	Safe.	Factor of safety.	Ultimate.	Safe.	Factor of safety.
Cast Iron . .	7·00	1·50	4·7	42·00	6·00	7·0
Wrought Iron	22·00	5·00	4·4	19·00	5·00	3·8
Steel * . . .	32·00	6·50	4·9	32·00	6·50	4·9
Brick . . .	0·12	—	—	0·25	0·05	5·0
Blue ditto .	—	—	—	1·00	0·20	5·0
Slate . . .	4·30	—	—	5·00	0·50	10·0
Granite . .	—	—	—	3·00	0·50	6·0
Sandstone .	—	—	—	1·00	0·20	5·0
Fir † . . .	5·00	0·75	6·6	2·75	0·50	5·5
Ash † . . .	7·00	1·00	7·0	4·00	0·75	5·3
Oak † . . .	5·00	0·75	6·6	3·00	0·50	6·0
Teak . . .	6·00	1·00	6·0	4·50	0·75	6·0
Cement ‡ (neat)	0·12	—	—	1·00	0·20	5·0
Ditto (2 to 1)	0·04	—	—	0·50	0·10	5·0

5. In calculating the strength of roofs, floors, and bridges, the loads usually allowed for are as follows :—

Pressure of wind on bridges under the control of the Board of Trade, or in very exposed situations . . . . . } 56 lbs.§ per square foot.

---

* The tensile strength of steel for ship plates according to Lloyds should be not less than 27 nor more than 31 tons per sq. in. with 20 P. ct. elongation in 8 ins. and for L.1.' not less than 27 nor more than 33 tons with 16 P. ct. elongation.

The plates should be capable of being bent to an inside radius of 1½ times their thickness when heated to a low cherry red, and cooled in water of a temperature of 28° C.

† The resistance of fir to shearing is ·25 tons, of ash ·50 tons, and of oak 1·00 tons, per sq. in. ultimate; or of ·05, ·10, and ·20 tons, respectively, safe.

‡ Portland Cement, set 30 days.

§ Inclined to the horizon at an angle of 30°.

Pressure of wind on roofs and bridges which are not in exposed situations	40*	lbs.	per square	foot.
Slating	6	,,	,,	,,
Plain Tiling	18	,,	,,	,,
Pan	9	,,	,,	,,
Glass, Zinc, or Corrugated Iron	4	,,	,,	,,
1 in. Slate boarding	3	,,	,,	,,
2½ in. × 3¼ in. Rafters laid 15 ins. centre to centre, including slate battens	2	,,	,,	,,
Live load on house floors and foot bridges	1	cwt.	,,	,,
Live load on railway passenger platforms	1½	,,	,,	,,
Live load on warehouse floors	3†	,,	,,	,,
Metalling and Ballast . . .	1¼	,,	per cubic	foot.
Permanent Way, single line, 84 lbs. rail, and 10 ins. × 5 ins. sleepers, 4 ft. 8½ ins. gauge exclusive of ballast—chair road	1	,,	,, lineal	,,
Roadway Bridges, moving load	1‡	,,	,, square	,,

Railway Bridges—moving load, single line.

For spans up to 16 ft. . .	16	tons	at centre,
,, from 30 ft. to 100 ft.,	1½	,,	per foot run,
,, above 100 ft. . .	1½	,,	,, on 100 ft.,
	¾§	,,	,, on the rest.

* Inclined to the horizon at an angle of 30°.

† 2 cwt. for grain sheds.

‡ Provision must be made for a traction engine. The 15 ton traction engine shown in Fig. 1, is the heaviest in common use.

§ 1½ tons throughout, if under Board of Trade control.

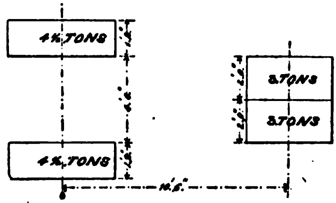


Fig. 1.

The maximum weight of a locomotive engine may be taken at 45 tons,* distributed over a length of 30 ft., and of this 16 tons may be on each of two axles placed 8 ft. centre to centre. With its tender the engine would weigh 70 tons distributed over 50 ft. In taking the loads on a bridge provision should be made for two engines and tenders equal to 100 ft. lineal at 1½ tons per foot, and ¾ of a ton per lineal foot for the train following, and this has been done in the foregoing table.

6. In foundations the following pressures may be safely put on a superficial foot:—

Rock	13	tons.
Chalk	4	"
Solid Blue Clay and Gravel . . .	3 to 6	"
London Clay	2	"
12 in. × 12 in. piles well driven . .	20 to 30	"

* Min. Inst. Civil Engineers, vol. lxxxii., p. 348.

The author is indebted to Messrs. Hunt, Hanbury, and Rowlandson for the following maximum weights of engines on their respective railways:—

Railway.	Length (buffer to buffer).	Total Weight.	Load on one axle.
Lancs. and Yorks. .	36·7 feet.	56·7 tons.	17·5 tons.
Metropolitan . . .	31·8 "	46·6 "	18·6 "
Mersey	——	51·0 "	17·0 "
Board of Trade . .	34·7 "	52·2 "	17·4 "

He is also indebted to Major-General Hutchinson, R.E., for the following equivalent distributed loads which are calculated from the engine which is designated above "Board of Trade."

Span in feet.	Rolling Load per foot run.
10 to 16	3·47 tons to 2·26 tons.
16 " 20	2·26 "
20 " 40	2·26 " " 1·74 "
40 " 55	1·74 " " 1·59 "
55 " 100	1·59 " " 1·57 "

Examples.

1. A cast iron cistern is 2 ft. long, 1 ft. 6 ins. wide, and 1 ft. 6 ins. deep over all. If the metal is $\frac{3}{8}$ in. thick, find the weight, and also that of the water contained in it, when its surface is $2\frac{5}{8}$ ins. below the top.

Answer, Cistern, 1 cwt. 3 qrs. 0 lbs.
Water, 1 cwt. 3 qrs. 21 lbs.

2. Will the above float if it is empty, and how much of it will be out of the water?

Answer, $5\frac{1}{2}$ ins.

3. Give the weight of a pitch pine balk 14 ins. square and 30 ft. long.

Answer, 13 cwt. 1 qr. 27 lbs.

4. If a square piece of elm be floated in fresh water, how much will be immersed?

Answer, ·544.

5. What thickness of wrought iron must be fixed to the bottom of this to cause it to float in salt water with 1 in. not immersed, the timber being 12 ins. deep?

Answer, ·71 in.

6. What must the length of a girder be for the changes of temperature to which we are liable in this climate to lengthen it 1 in.?

Answer, 166 ft. 8 in.

7. What loads may be safely put on wrought iron bars $1\frac{1}{4}$ in. 1 in. and $\frac{3}{4}$ in. in diameter?

Answer, 6·1, 3·9 and 2·2 tons respectively.

8. The weight of the superstructure of one span of a viaduct together with the moving load amounts to 100 tons. What size should the foundation of one of the piers be if it is of brickwork, 18 ft. long, 3 ft. 9 ins. wide and 20 ft. high, and rests on blue clay?

Answer, The 18 ft. × 3 ft. 9 ins. base would be sufficient as the pressure per sq. ft. with this would be only 2·48 tons.

9. There is halt a yard of ballast per yard run on a rail-

way bridge. Give the weight per lineal foot of the dead and live loads exclusive of the weight of the structure itself for the following spans—single line:—

16 feet, 20 feet, 30 feet, and 200 feet.
Answer, 2·33 tons, 2·25 tons, 1·83 tons, and 1·83 tons.

10. A water tank is 6 ft. deep; find the pressure per lineal foot against its side, and the depth below the surface at which its resultant acts, the tank being full.

Answer, $\frac{1}{2}$ ton at 4 ft. below the top.

CHAPTER II.

SIMPLE STRAINS IN GIRDERS AND FRAMED STRUCTURES.

7. *If a weight rests through the medium of a beam on two abutments it is supported by their reactions, and the sum of these is equal to the weight.*

If the weight is central, the reaction of each abutment is equal to half the weight; if not, it may be determined by the principle of moments in the following manner :—

In Fig. 2 let W be the weight acting at c, and let it be supported by the reactions R_1, R_2 at a and b.

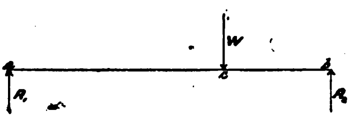


FIG. 2.

By taking moments about b we have :—

$$R_1.ab = W.cb, \text{ i.e. } R_1 = \frac{cb}{ab} W.$$

And by taking moments about a we have :—

$$R_2.ab = W.ac, \text{ i.e. } R_2 = \frac{ac}{ab} W.$$

Following from which we see that :—

$$R_1 + R_2 = \frac{cb + ac}{ab} W,$$

$$= W.$$

Let the depth of the beam be h, the area of the flange in square inches a, and the resistance of the metal to tension or compression, in tons per square inch when W is in tons, s.

Then we have, by taking moments at c about either flange:—

$$R_1.ac \quad = s.ah,$$

or, putting M for ah . . . . . $= s.M.$

M is called the moment of resistance of the beam. For plain girder sections, as in Fig. 3, in which no account is taken of the web, it is simply the area of either flange multiplied by the depth between the flanges. More complicated sections will be considered in Chapter iv., but we may state here that the general value of M is $a_1 s_1 h_1 + a_2 s_2 h_2 + \quad . \quad . \quad . \quad + a_n s_n h_n$; the area of the cross section being divided up into parts, and each taken separately, $a_1, a_2,$ . . . being the area of each part, $s_1, s_2,$ . . . the strain per unit in it, and $h_1, h_2,$ . . . the distance apart of the corresponding tensile and compressive members.

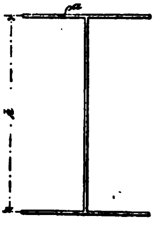


Fig. 3.

8. If the weight is central $R_1 = \frac{W}{2}$, also $ac = \frac{ab}{2}$ and this we will call $\frac{l}{2}$.

Hence we may write $\frac{W}{2}.\frac{l}{2} \quad = sM.$

Halfway between the centre of the bearing and the abutment, we have

$$\frac{W}{2}.\frac{l}{4} \quad = (sM)',$$

so, $$(sM)' \; = \; \frac{sM}{2}.$$

Now if the moment of resistance of the section is proportional to the strain s will be constant, but if the section of the girder is the same throughout s will vary.

Thus we have $$\frac{W}{2}\frac{l}{4} = sM' \text{ or} = s'M.$$

9. If the beam carries an equally distributed load W, in taking moments about the centre we may consider half the weight to act at d, and half at e, Fig. 4, these points being $\frac{l}{4}$ from each abutment, and we have :—

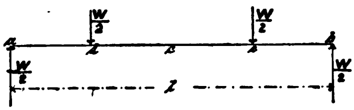


FIG. 4.

$$\frac{W}{2}\cdot\frac{l}{2} - \frac{W}{2}\cdot\frac{l}{4} = sM,$$

i.e. $$\frac{W}{2}\cdot\frac{l}{4} = sM,$$

Again, taking moments at d or e,

$$\frac{W}{2}\cdot\frac{l}{4} - \frac{W}{2}\cdot\frac{l}{8} = (sM)',$$

i.e. $$\frac{3}{32}\ W.l = sM' \text{ or} = s'M.$$

10. It is convenient to show the amount of the moments by a diagram as is done in Olander's "Girder Construction," a work which we cannot too strongly recommend to the student, the plates being of real practical value.

If s is constant, the variable values of M may be plotted off the line ab. Thus, in the case of a concentrated load, Fig. 2, if the depth of the girder is constant, the necessary sectional area at any point can be

obtained by scale from the diagram Fig. 5, the height $o.c$ being marked off to any convenient scale equal to a.

It will be seen with a very little consideration that the lines oa and ob will be straight, so that midway between c and a the sectional area necessary will be $\frac{a}{2}$, and so on for any other point.

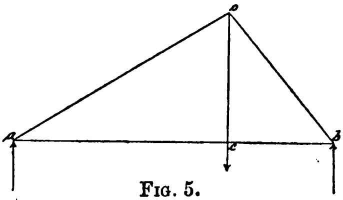


FIG. 5.

In the case of the distributed load, Fig. 4, the line $a\,o\,b$ is a parabola, as can be easily proved. If the student has not, however, studied Conic Sections, he must plot the curve by taking moments at a number of points.

Thus at c the height oc will be equal to $\frac{1}{8}\cdot\frac{W.l}{s.h}$, and at d and e . . . $\frac{3}{32}\cdot\frac{W.l}{s.h}$.

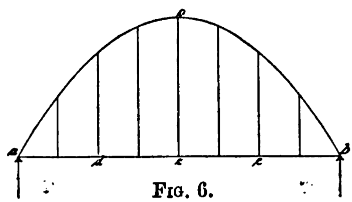


FIG. 6.

11. With a cantilever or beam fixed at one end and loaded at the other $W.l = sM$, and for a distributed load $W\cdot\frac{l}{2} = sM$.

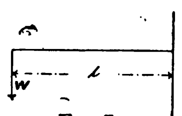


FIG. 7.

12. A beam fixed at both ends is really composed of two cantilevers and a girder. The strains for a central load in such a beam act as in Fig. 8, and for a distributed load as in Fig. 9. Taking one flange, say the top, for the purpose of illustra-

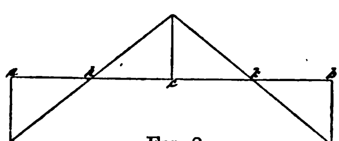


FIG. 8.

tion, it will be seen that from *a* to *h* and *b* to *k* the strain is tensile, and from *h* to *k* compressive, and that at *h* and *k* there is no strain at all.

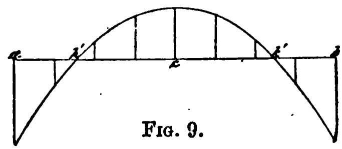


FIG. 9.

A beam continuous over several spans may be regarded as fixed at the ends for each span, and is called a continuous girder.

13. *If a framed structure, such as* a, b, c, *Fig.* 10, *supports a weight* W *at* c, *and is itself supported at* a *and* b, *the strains on the various members may be calculated by means of the triangle of forces.*

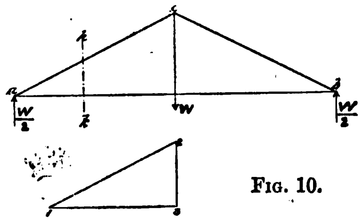


FIG. 10.

At the point *a* we have the vertical upward resistance of $\frac{W}{2}$, and the strains in *ca*, *ab* which maintain equilibrium.

Draw the triangle *1. 2. 3*, so that its sides are severally parallel to *ac*, *cW* and *ba*. Then by the triangle of forces *1. 3 : 3. 2 : 2. 1 : :* strain in *ab :* upward resistance $\frac{W}{2}$ *:* thrust *ca*; and the forces act in the directions *1* to *3*, *3* to *2*, and *2* to *1*, so that *ca* is in compression and *ab* in tension.

Now since the upward thrust $\frac{W}{2}$ represented by *3.2* is known, the thrust *2.1* and tension *1.3* can be calculated; but it is the simpler plan to plot the triangle to scale and measure the strains.

Thus, if *3.2* and *2.1* measure 1 in. and 2 ins. respectively, and if $\frac{W}{2}$ is 10 tons, the thrust *ca* will be 20 tons.

This method of calculation is called the graphic method; it is quicker, and there is in it less chance of a grave error than there is in the more mathematical method.

14. There are three ways by which the strains in the above figure may be calculated :—

The first is by the *graphic method*, as shown above.

The second by means of *moments*, the strain in *ab* being calculated by taking moments about *c*, and that in *ac* about any point in *ab*.

The third by the *method of sections;* for if a section *hk* is taken, it is clear that for equilibrium all the resolved parallel forces acting in one direction must equal all those acting in the contrary.

15. As an example we will take the roof truss shown in Fig. 11, and calculate the strains by each of the three methods :—

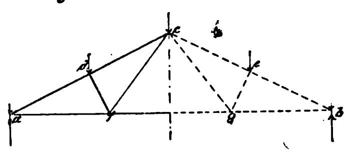


FIG. 11.

Call the span *ab*, *l*, and let the height be $\frac{l}{4}$ and the equally distributed load *W*, and let $ad = dc$.

The equally distributed load will be supported by a vertical upward resistance of $\frac{W}{2}$ at each abutment *a* and *b*, and the load may be assumed to act as follows :—

$\frac{W}{8}$ at *a*, $\frac{W}{4}$ at *d*, $\frac{W}{4}$ at *c*, $\frac{W}{4}$ at *e*, and $\frac{W}{8}$ at *b*.

We will first proceed by the graphic method, and to simplify the figure and reduce the labour will only consider half of the truss.

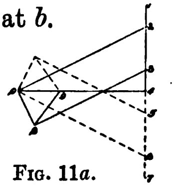


FIG. 11*a*.

Draw the vertical line *1.7* and divide it so that *1.2* will represent $\frac{W}{8}$, *2.3*, $\frac{W}{4}$, *3.5*, $\frac{W}{4}$, etc.

Bisect *3.5* in *4*, draw the horizontal line *4.0* through it, and draw *0.2* and *8.3* parallel to *a.c*, *0.8* parallel to *d.f*, and *8.9* parallel to *f.c.*;—

Taking first the strains at *a*;—

We have a vertical upward resistance of $\frac{W}{2}$ represented by *4.1*, a downward vertical pressure of $\frac{W}{8}$ represented by *1.2*, and a thrust along *da* and a pull along *ab* represented respectively by *2.0* and *0.4*, the amounts of which can be ascertained by scaling from the figure 11*a*.

Next taking the point *d*;—

We have an upward thrust along *ad* represented by *0.2*, a downward pressure of $\frac{W}{4}$ represented by *2.3*. and thrusts along *cd* and *fd* represented by *3.8*. and *8.0*. respectively.

Again at *f*;—

We have along *fa*, *df*, *fc*, and *fg* strains represented by *4.0*, *0.8*, *8.9*, and *9.4* respectively.

And at c;—

Along *cf* and *dc* there are strains represented by *9.8* and *8.3* which, combined with half the weight at *c* or $\frac{W}{8}$ represented by *3.4*, are resisted by a horizontal thrust of *4.9*, which is equal to the strain in *fg*.

By scaling from Fig. 11*a*, we find that the strains in the different members are as follows:—

In *ad*	a thrust	represented by	*0.2*	or	$+\cdot84\ W$
" *dc*	"	"	*8.3*	"	$+\cdot72$ "
" *df*	"	"	*0.8*	"	$+\cdot22$ "
" *af*	a tension	"	*0.4*	"	$-\cdot75$ "
" *fg*	"	"	*9.4*	"	$-\cdot50$ "
" *fc*	"	"	*8.9*	"	$-\cdot25$ "

and the external forces are the vertical upward pres-

sure of $\frac{W}{2}$ at a, the horizontal thrust of ·50 W at c, and the pressures of $\frac{W}{8}, \frac{W}{4}, \frac{W}{8}$ at a, d and c respectively.

16. We will next proceed by the principle of moments, and taking first the moments round the point c, we have—

tons.

$$\frac{W}{2}\cdot\frac{l}{2} - \frac{W}{8}\cdot\frac{l}{2} - \frac{W}{4}\cdot\frac{l}{4} - \text{strain in } fg\,\frac{l}{4} = 0,$$

Hence the strain in fg or *9.4* $= W\dfrac{(1-\frac{1}{4}-\frac{1}{4})\frac{l}{4}}{\frac{l}{4}},$

$= \cdot 50\ W.$

Also $\frac{W}{4}\cdot\frac{l}{4} - \text{strain in } df \times cd$ $=0,$

Hence the strain in df or *0.8* $= \dfrac{\frac{W}{4}\cdot\frac{l}{4}}{\frac{l}{8}\sqrt{5}},$

$= \cdot 223\ W.$

Next taking moments about the point f, we have—

$$\frac{W}{2}\cdot\frac{5l}{16} - \frac{W}{8}\cdot\frac{5l}{16} - \frac{W}{4}\cdot\frac{l}{16} - \text{strain in } cd.\frac{\sqrt{5}}{16}l = 0,$$

Whence the strain in cd. $= \cdot 726\ W;$

and $\frac{W}{2}\cdot\frac{5l}{16} - \frac{W}{8}\cdot\frac{5l}{16} - \text{strain in } ad.\ \frac{\sqrt{5}}{16}l = 0,$

Whence the strain in ad. $= \cdot 84\ W;$

or we might say—

$$\text{Strain in } ad. \frac{\sqrt{5}}{16} l - \text{strain in } cd. \frac{\sqrt{5}}{16} l - \frac{W}{4} \cdot \frac{l}{16} = 0,$$

or
$$(\cdot 84 - \cdot 726)\, W \frac{\sqrt{5}}{16} l = \frac{W}{4} \cdot \frac{l}{16},$$

which shows a very near agreement.

Again taking moments at d—

$$\frac{W}{2} \cdot \frac{l}{4} - \frac{W}{8} \cdot \frac{l}{4} - \text{strain in } af. \frac{l}{8} = 0,$$

Whence the strain in af $= \cdot 75\ W.$

Also,

$$\cdot 75\ W \frac{l}{8} - \cdot 50\ W \frac{l}{8} - \text{strain in } fc. \frac{l}{8} = 0,$$

Whence the strain in fc $= \cdot 25\ W.$

17. Lastly by the method of sections.

Draw a vertical line between a and d, and resolve the strain in ad horizontally and vertically by means of the parallelogram of forces.

Since the vertical component must be equal to the weights which have to be transmitted through it, we have;—

$$\text{The strain in } ad = \frac{\frac{W}{4} + \frac{W}{8}}{\sin dab} = \frac{3}{8} \cdot \frac{W}{\sin dab},$$

$$= \frac{3}{8} \cdot \frac{W}{\frac{1}{\sqrt{5}}} = \cdot 84\ W.$$

And " " $af = \text{strain in } ad. \cos dab,$

$$= \frac{3}{8} \cdot \frac{W}{\frac{1}{\sqrt{5}}} \cdot \frac{2}{\sqrt{5}} = \cdot 75\ W.$$

It is not necessary to exemplify this method further, as the strains in the other members can be calculated in exactly the same way.

EXAMPLES.

1. If the span of the cross girders of a railway bridge is 11 ft. and the rails are 5 ft. centre to centre, each rail being thus 3 ft. from the nearest support, find the value of M, s being 5 tons and the weight on each rail 8 tons.
Answer, 4·8.

2. If the depth is 12 inches, give the sectional area.
Answer, 4·8.

3. If the span is 16 ft. and each rail 5 ft. 6 ins. from the support, find M and a.
Answer, $M=8·8$, $a=8·8$.

4. If the span is 26 ft., and the line double, the rails being thus respectively 5 ft. and 10 ft. from the supports, find the values of M and a at each rail, the depth being 2 ft.
Answer, $M=24$ ins. and 16 ins, $a=12$ ins. and 8 ins.

5. For a cantilever loaded at the end, prove that $a=\frac{Wl}{sd}$

6. " " " equally " $a=\frac{Wl}{2sd}$

7. " girder " at the centre " $a=\frac{Wl}{4sd}$

8. " " " equally " $a=\frac{Wl}{8sd}$

Where a is the sectional area in square inches,
" W " the load in tons,
" s " the resistance of the metal in tons per square inch,
" l " the span in feet,
" d " the depth in feet.

9. A load of 8 tons is 2 ft. from one support and 14 ft. from the other; find the reactions of the supports, and the value of M at the load and at the centre of the span.
Answer, 7 tons and 1 ton, 2·8 ins. and 1·6 ins.

10. The load on a girder is one ton per foot run, the span 25 ft. and the depth 2 ft. 6 ins.; find the necessary sectional area.
Answer, $6\frac{1}{4}$ ins.

11. Find the sectional area for a span of 60 ft. and depth of 6 ft., when there are two girders carrying a double line of railway.
Answer, 30 ins.

12. A rail weighing 69 lbs. per yard and 16 ft. 6 ins. long rests on a fulcrum 15 ins. from one end, and sustains a weight of 1 cwt. at 3 ins. from the other; find the force exerted 12 ins. from the fulcrum.

Answer, 38·72 cwt.

13. In the last question if the force is a ton, how far will it act from the fulcrum, the 1 cwt. being removed?

Answer, 14·23 ins.

14. In the last question find the position of the weight which will produce a pressure of (*a*) 3 tons and (*b*) 4 tons, at a distance of 6 ins. from the fulcrum.

Answer, (*a*) 1 cwt. at 6 ft. 3·43 ins., and (*b*) 2 cwt. at 8 ft. 1·71 ins. from the fulcrum.

15. A cantilever, 12 ins. deep and 12 ft. long, sustains an equally distributed load of 3 tons; find the sectional area required if the resistance of the metal is taken as 5 tons per square inch. *Answer*, 3·6 ins.

16. Girders 36 ft. span, 12 ft. apart and 3 ft. deep, carry (*a*) the roof of a passenger station, and (*b*) a warehouse floor. Find the sectional area in each case.

Answer, (*a*) 2·31 ins., *taking the load as 40 lbs. per sq. ft.*
(*b*) 19·44 ins.

17. Draw a diagram of the strains on the roof of which one of the principals, which are 12 ft. apart, 30 ft. span and 7 ft. 6 ins. high, is shown in Fig. 11, it being acted on by a pressure of 40 lbs. per square foot at an angle of 30° to the horizon, in addition to a vertical dead load of 14 lbs. per square foot.

18. The conditions being the same as in the last question, draw a diagram of the strains, the span being 40 ft. and the height 8 ft. 6 ins. from the apex to the tie bar and 10 ft. to the supports.

19. A lattice girder 100 ft. span and 10 ft. deep is divided into 10 equal bays by verticals which take the compressive strains, the diagonals taking the tensile. The dead load being 15 cwt. per lineal foot, trace the progress of a rolling load equal to 30 cwt. per lineal foot on to and over the bridge.

(N.B. *The loading is as for one of two girders carrying a double line of railway.*)

20. Find what height of salt water pressing against it will cause a brick wall, 3 ft. thick and 6 ft. high, to revolve about a point 1 ft. from the outer edge.

Answer, 4 ft. 6½ ins.

21. To what height can a brick wall 18 ins. thick be carried so that a horizontal wind pressure of 40 lbs. per square foot will not overturn it on its edge—(*a*) neglecting the tenacity of the mortar, and (*b*) assuming the elasticity to be perfect, and the maximum tensile resistance of the mortar to be 10 lbs. per square inch.

Answer, (*a*) 6 ft. 3½ ins., (*b*) 11 ft. 2 ins.

22. If a wall 13½ ins. thick and 12 ft. high has cantilevers fixed to it at the top, to what distance can they be carried in order that the line of action of the resultant may fall within the middle third of the wall, the resolved vertical pressure on the roof being 56 lbs. per square foot?

Answer, 2 ft. 10 ins.

23. Two steel girders 6 ft. deep carry a single line of railway over a span of 60 ft. If the sectional area is constant throughout, give an elevation showing the varying depth.

24. Give the necessary sectional area at each 10 ft. in the last case, if the depth is constant.

Answer. At 10 ft. from the abutment, 6·4 ins., at 20 ft., 10·25 ins., and at the centre, 11·5 ins.

25. The iron cantilevers of a verandah are constructed as in Fig. 12, and exposed to an equally distributed vertical pressure of 56 lbs. per square foot. Give the sectional areas of the flanges at the points *A.A*, *B.B*, and *C.C*, the cantilevers being 10 ft. apart and not bolted down.

Answer, 3·6 at *A.A*, 3·6 and 4·2 at *B.B*, and 3·6 at *C.C*.

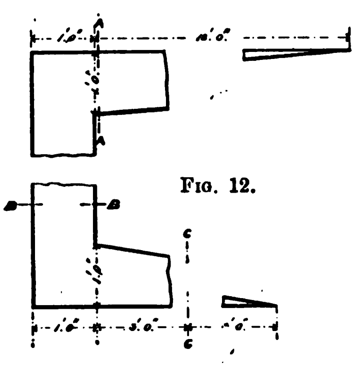


FIG. 12.

26. The driving axles of a locomotive are 8 ft.

centre to centre, and the weight on each 16 tons; find the required moment of resistance of one of the rails, if the metal is strained to 10 tons per square inch, and the upward resistance of the sleepers is considered as equivalent to an equally distributed pressure. *Answer*, 9·6 ins.

27. If in the last question the span is taken as 6 ft., 1 ft. at each end being considered as a cantilever, what is the moment?

Answer, 5·4 ins.

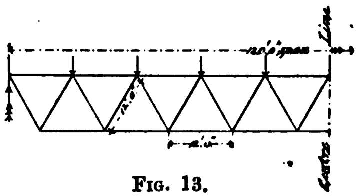


Fig. 13.

28. The principals of a roof are 50 ft. span, and are supported by the iron warren girders of 120 ft. span, shewn in Fig. 13. Give the strains on the various members, the load being taken at 56 lbs. per square foot acting vertically.

Answer	1st Bay	2nd Bay	3rd Bay	4th Bay	5th Bay.
Diagonals	$15\frac{1}{2}$	$12\frac{1}{4}$	$8\frac{3}{4}$	$5\frac{1}{4}$	$1\frac{3}{4}$
Top flange	8	$21\frac{1}{2}$	32	39	$42\frac{1}{2}$
Bottom „	$15\frac{1}{2}$	$27\frac{1}{2}$	$36\frac{1}{2}$	$41\frac{1}{2}$	$43\frac{1}{4}$

29. A cofferdam sustains the pressure of a head of 24 ft. of water, and is supported every 10 ft. at 4 ft. from the top by props which are inclined at an angle of 60° to the horizontal. Find the pressure along each (*a*) if the water is fresh, and (*b*) if it is salt.

Answer, (*a*) 64·3 tons, and (*b*) 65·8 tons.

30. If the inclination is $\frac{\pi}{4}$, find the pressure in the first case. *Answer*, 45·5 tons.

31. A rail 26 ft. long, weighing 67·2 lbs. per yard, rests on a fulcrum near one end and supports a weight of a ton at a distance of 2 ft. from the fulcrum. Find the distance of the latter from the end, when the rail is just balanced.

Answer, 5 ft. $3\frac{1}{16}$ ins.

32. In the last case what pressure would 2 cwt. placed 16 ft. from the fulcrum exert at a distance of 6 ins. from it, if the one ton weight was removed? *Answer*, 7·2 tons.

CHAPTER III.

STRAIN OUT OF CENTRE.—THIN TUBES.—THICK TUBES.—RIVETED JOINTS.—TORSION.

Strain out of Centre.

18. *A force P acts as shewn in Fig.* 14, *and is resisted by four parallel and equally spaced forces* p_1, p_2, p_3, p_4, *through the medium of a rod ab, which will not bend; it is required to find the values of* p_1, p_2, p_3, p_4 *in terms of P.*

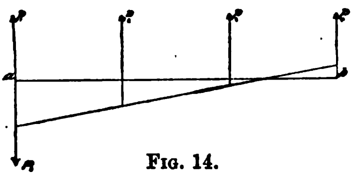


Fig. 14.

We have $\quad p_1 + p_2 + p_3 + p_4 = P \ldots\ldots\ldots (1)$

Also $\quad 3p_1 + 2p_2 + 1p_3 = 3P \ldots\ldots\ldots (2)$

And (22) $\quad p_1 - p_2 = p_2 - p_3 = p_3 - p_4$

i.e. $\quad p_1 + p_3 = 2p_2 \ldots\ldots\ldots (3)$

and $\quad p_2 + p_4 = 2p_3 \ldots\ldots\ldots (4)$

By solving these equations we obtain

$$p_1 = 7p_3 = \tfrac{7}{10} P,$$
$$p_2 = 4p_3 = \tfrac{4}{10} P,$$
$$p_3 = \tfrac{1}{10} P,$$
$$p_4 = -2p_3 = -\tfrac{2}{10} P.$$

It will be observed that a compressive strain at p_1

induces a tensile at p_4, a result which has an important bearing on the strength of brick and stone arches and reservoir walls.

19. *To find the neutral point, i.e. the point where the strain is zero, when P is resisted by an infinite number of parallel forces.*

Let p be the unit strain opposite P.

This strain will decrease from a to b (Fig. 15), where it will become zero, and so again to c, where it will be negative, b being taken as the neutral point.

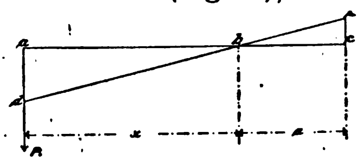


FIG. 15.

Put $ab = x$ and $bc = a$

Then the mean unit strain between a and b will be $\frac{p}{2}$, and that between b and c will be $\frac{a}{x}\frac{p}{2}$.

$$\text{Hence} \qquad P = \frac{p}{2}x - \frac{ap}{2x}a \quad = \frac{p}{2x}\left\{x^2 - a^2,\right\},$$

$$\text{or} \qquad \frac{P}{p} = \frac{x^2 - a^2}{2x} \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{Also } P\left(x + \frac{2}{3}a\right) = \frac{p}{2}x\left(\frac{2}{3}x + \frac{2}{3}a\right) = \frac{p}{3}x\left(x + a\right),$$

$$\text{or} \qquad \frac{P}{p} = \frac{x(x+a)}{3x + 2a} \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2)

we get $x^3 - 3a^2x - 2a^3 = 0$,

And the solution $x = 2a$ satisfies this.

20. It is clear that for the strain at c to be zero the force P must be applied at a point which is $\frac{2}{3}ac$ from c,

for the centre of the parallel forces decreasing from p at a to 0 at c coincides with the centre of gravity of the triangle acd, Fig. 16.

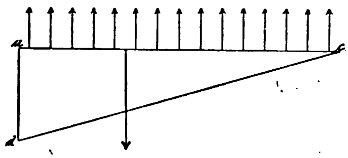


FIG. 16.

21. From this it will be seen that with materials which offer but a small resistance to tension, a force must not be applied outside the middle third of the width; and that when applied at one-third from the edge the strain at this edge will be at a maximum, while at the other edge it will be zero, and the maximum unit strain will be double the force divided by the area of the section.

In the case of an arch it is therefore necessary that under every condition of loading the line of resistance should be within the middle third; and in the case of a reservoir wall that it should be so placed, whether the reservoir is empty or full. When empty it is only the weight of the wall itself which has to be dealt with, so that it is only necessary that at every horizontal section a vertical line through the centre of gravity should be within the limits. When full in addition to the weight the moment of the water pressure has to be allowed for.

All materials offer some resistance to tension, and in some cases this may be taken account of; but it is generally better to act on the above principle and let the resistance to tension be considered as so much safety.

22. It will be noticed in (18) that the forces p_1, p_2, . . . have been assumed to be in Arithmetical Progression. This is true within certain limits so far as experiment has revealed. All materials that have been experimented on up to a certain point, which is called the limit of elasticity and is in all cases very much higher than the working load, yield under both compression and tension in direct proportion to the strain.

Thin Tubes.

23. *To find the strains on a thin tube exposed to internal pressure.*

Let ab (Fig. 17) be the diameter of a cylinder in which the thickness of the metal is small compared with the diameter; it is required to find the strain on the metal caused by a force of p per unit of area acting at right angles to the surface.

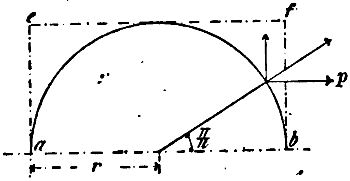


Fig. 17.

If the forces p are resolved horizontally and vertically, the sum of the vertical components will have to be resisted at a and b, that is to say, the strain at a will be half the sum of the vertical components. If there are n units of length in the circumference $a\ b$, each will be equal to $\frac{\pi r}{n}$, and the total vertical pressure will be

$$p\frac{\pi r}{n}\left\{\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \ . \ . \ . \ + \sin\frac{n\pi}{n}\right\}$$

and this becomes (sec. 1) $p.2r$, so that the strain at a will be $p.r$.

24. This result might also have been obtained in the following way:—

Let $cdef$ (Fig. 18) be a square of which the length of each side is $2r$.

Then the pressure against one side ef will be $p.\ 2r$, and this will be resisted by the tensile strains in ce and df, each of which will thus be pr; *i.e.* the strain at a will be pr, and at b, pr.

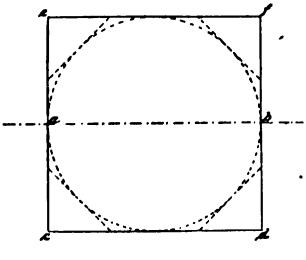


Fig. 18.

It is evident by the principles of hydrostatics that the strains at a and b will not be disturbed by altering the square into an octagon, as shown by dotted lines on the figure, and by again increasing the number of sides until eventually it becomes a circle.

Thick Tubes.

25. *To find the effective resistance of the metal in the case of a cylinder in which the thickness is considerable when compared with the diameter, as in the case of an hydraulic press pipe.*

Let D_1 and D be the external and internal diameters before strain, and $(D_1 + d_1)$, $(D + d)$ after strain.

There will always be the same quantity of metal in the ring whether it is strained or not, so that after reducing we have the equation

$$D_1^2 - D^2 = (D_1 + d_1)^2 - (D + d)^2,$$

$$\text{or} \qquad \frac{2D_1 + d_1}{2D + d} = \frac{d}{d_1}.$$

Now since d_1 and d are always very small compared with D_1 and D, we can put

$$\frac{D_1}{D} = \frac{d}{d_1}.$$

Let s_1 be the strain at the exterior, and s at the interior. Then d_1 will vary as $s_1 D_1$, and d as sD, hence

$$\frac{D_1}{D} = \frac{s D}{s_1 D_1},$$

$$\text{or} \qquad \frac{s}{s_1} = \frac{D_1^2}{D^2} = \frac{\frac{1}{D^2}}{\frac{1}{D_1^2}}$$

i.e. the strain varies inversely as the square of the diameter.

Thus, if the internal diameter is 10 and the external 20, *i.e.* if $D = 10$ and $D_1 = 20$, we have

$$\frac{s}{s_1} = \frac{\dfrac{1}{10^2}}{\dfrac{1}{20^2}} = \frac{4}{1}, \text{ or } s = 4s_1,$$

i.e. the strain on the inside is four times the strain on the outside.

To find the mean strain in terms of the maximum we must assume the thickness to be divided up into a number of small parts, say n in number.

Then the strain per unit

where the diameter is D is s or, $\dfrac{\dfrac{1}{D^2}}{\dfrac{1}{D^2}} s$

" " $D+1$ " $\dfrac{\dfrac{1}{(D+1)^2}}{\dfrac{1}{D^2}} s$

" " $D+n$ or D' " $\dfrac{\dfrac{1}{(D+n)^2}}{\dfrac{1}{D^2}}, s$

Hence the mean strain is

$$\frac{D^2 s}{n+1}\left\{\frac{1}{D^2} + \frac{1}{(D+1)^2} + \ldots + \frac{1}{(D+n)^2}\right\}$$

Putting $D=10$, $D+1=12$, . . . and $D+n=D'=20$, we get as the mean strain:—

$$\frac{10^2}{6} s \left\{\frac{1}{10^2} + \frac{1}{12^2} + \ldots + \frac{1}{20^2}\right\},$$

or $$s\left\{\frac{1 + \cdot 7 + \cdot 5 + \cdot 4 + \cdot 3 + \cdot 25}{6}\right\},$$

or $$\frac{s}{2}.$$

If therefore the maximum resistance of the metal is taken as 7 tons per square inch, the mean resistance will be only $3\frac{1}{2}$.

Riveted Joints.

26. In calculating the strength of riveted joints there are four points to be considered:—

1. *The sectional area of the plate after the rivet holes have been deducted must be sufficient to resist the tensile or compressive strain.*
2. *The number of rivets must be sufficient to resist the shearing strain on them.*
3. *There must be metal enough between the rivet holes to resist the shearing force, i.e., if t. is the thickness of the plate and a the distance between the holes,* 2*n.t.a.s. must be greater than the strain.*
4. *The metal must be thick enough to prevent the rivets tearing through it, i.e., if d. is the diameter of the rivets, n.d.t.s. must be greater than the strain; but in this case, owing to the support given on each side, s may be taken at* 10 *tons or even more.*

With respect to the first point, we have not only to consider the reduced sectional area, but also in the case of punched holes the effect of the punching on the material left; for the punch in passing through the plate injures the metal to some distance round the hole, this being especially the case with steel. By annealing the strength of the injured parts may be restored. Drilled holes in no way injure the metal, so that with steel it is often false economy to use the punch.

The mean of 9 experiments on $\frac{3}{8}$ in., $\frac{1}{2}$ in., and $\frac{5}{8}$ in. plates of Yorkshire iron, 8 in. wide with 4 holes each ·85 in. diameter, gave a loss of strength of 13 per cent.*

Now the material left was $8 - 4 \times {\cdot}85$ in. $= 4{\cdot}60$ in.,

---

* Box, 14.

and 13 per cent. of this is ·60 in., which divided by 4 gives ·15 in. per hole; so that instead of allowing for four holes of ·85 in. in diameter, we must take four of ·85 + ·15 in. *i.e.* of 1 in. diameter.

It follows from this that the injured metal extends to a distance of about $\frac{1}{16}$ in. beyond the hole, so that a $\frac{3}{4}$ in. hole should be punched and drilled out to $\frac{7}{8}$ in. for a rivet of this size, to avoid injuring the metal.

With steel the injury caused by punching is so considerable that drilled holes should always be employed, or the plates should be annealed after punching, for this entirely restores the strength of the injured parts.

Theoretically no deduction need be made for the holes when the strain is compressive, but it is difficult to believe that a rivet will so thoroughly fill the vacant space as to take up the strain equally with the plate; it is therefore better to deduct, especially as the resistance of wrought iron to compression is less than to tension and, as will be seen hereafter, the elasticity the same, so that the putting less metal in the compressive members of any structure would set up complex strains.

For the rivets we have the formulæ:—

$$\mathrm{R} = n\pi\left(\frac{d}{2}\right)^2 s \text{ when they are in single shear,}$$

$$\text{and } \mathrm{R} = 2n\pi\left(\frac{d}{2}\right)^2 s \quad \text{,,} \quad \text{,,} \quad \text{double} \quad \text{,,}$$

where n is the number, d the diameter, and s the safe resistance of the metal, or 5 tons per square inch.

The resistance of iron to shearing is the same as to tension, or about 22 tons per square inch. Probably owing to the injury they receive in the process of riveting steel rivets are no stronger than iron, in spite of its being generally admitted that the shearing and tensile resistance are the same.

In addition to the resistance they offer to shearing, the rivets hold the plates together by their grip. Three

experiments were made with a $\frac{7}{8}$ in. rivet, and the resistance due to the friction caused by the grip was:—

When the length of the rivet was $1\frac{1}{4}$ ins. . . $4\frac{3}{4}$ tons,*
" " " $1\frac{7}{8}$ " . . $5\frac{1}{2}$ "
" " " $2\frac{1}{8}$ " . . 8 "

In calculating the strength of a joint this is not usually taken into account; but it may be considered as so much safety to cover the loss of strength caused by the various rivets not all taking up the strain equally, on account of imperfect workmanship.

Torsional Strain.

27. The resistance of any material to torsion is dependent on its resistance to shearing. Thus if a force W (Fig. 19), be applied with a leverage l to a shaft of radius r, we have where s is the resistance of the metal to shearing:—

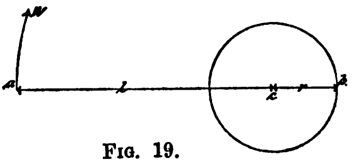


Fig. 19.

$$Wl = \{2\,\pi r.r + 2\pi\,(r-1)(r-1) + \ldots + 2\,\pi\,1.1\}s,$$
$$= 2\,\pi s\{r^2 + (r-1)^2 + \ldots + 1^2\},$$
$$= 2\,\pi s\,\frac{r\,(r+1)(2r+1)}{6},$$
$$= 2\,\pi s\,\frac{r^3}{3}, \quad \text{since 1 is small compared with } r.$$

With cast iron we may put $s = 9$ tons per square inch, and we get:—

$$Wl = 18{\cdot}85r^3 = 2{\cdot}36d^3 = (1{\cdot}33d).^3$$

For wrought iron s may be put at 18 tons, and for

* Box. 17.

steel at 27; so that the resistances of cast iron, wrought iron, and steel are as 1 : 2 : 3, and this is borne out by experiment.

28. In order that the material may not be strained beyond the limit of elasticity, it is necessary to proceed in another way. Thus, taking the case of wrought iron, it is evident that if the shaft is strained to the extent of say 10 tons per square inch at the circumference, it will only be strained to the extent of 5 tons per square inch midway between the circumference and the centre, for the fibres will be lengthened only to half the amount of those at the circumference, which shows that only half the strain is acting.

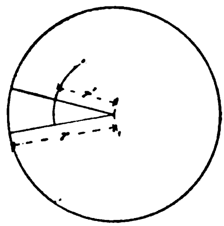


FIG. 20.

To find the resistance we have therefore

$$Wl = 2\,\pi r.\,10r + 2\,\pi\,(r-1)\left(\frac{r-1}{r}\right)10\,(r-1) + \ldots$$

$$+ 2\,\pi\,1\,.\frac{1}{r}\,.\,10\,.\,1,$$

$$= \frac{2\pi\,.\,10}{r}\,\{r^3 + (r-1)^3 + \ldots + 1^3\}$$

$$= \frac{2\pi\,.\,10}{r}\,\frac{r^2(r+1)^2}{2^2}$$

$$= \pi\,.\,10\,.\,\frac{r^3}{2}\ \text{for } r \text{ is large in comparison with } 1.$$

$$= 15{\cdot}7\,r^3 = 2d^3,\ \text{nearly},$$

$$\text{Or}\quad = \pi s\,\frac{r^3}{2}.$$

$$\text{Whence } s = 2\,\frac{Wl}{\pi r^3}.$$

The torsional elasticity can be most conveniently measured by the distance moved through by the weight W at the point a (Fig. 19).

If c is put for the fraction denoting the twist at b for a unit of the length of the shaft for a strain of a ton per square inch, the total twist at b, a being the length of the shaft, will be:—

$$s.a.c. \quad i.e. \quad \frac{2\,Wl}{\pi r^3}.a.c.$$

and at a. $$\frac{l}{r}\cdot\frac{2Wl}{\pi r^3}\cdot a.c. \quad ,, \quad \frac{2\,Wl^2}{\pi r^4}\,a.c.$$

The following is the value of c for certain materials as obtained by Mr. Bevan—

Wrought Iron and Steel . .	·0002
Cast Iron	·0004
Boxwood and Teak	·0133
Ash, Beech, Birch, Larch, Oak, and Pear	·0224 to ·0175
Deal, Elm, Fir, and Pine . .	·0330 to ·0243

EXAMPLES.

1. A load of a ton per lineal foot rests on a brick wall 18 ins. wide at 6 ins. from one edge; find the pressure per square inch at each edge and at the centre.

Answer 0, 10·4 and 20·8.

2. If the load is at the centre, give the pressures.

Answer, 10·4, 10·4 and 10·4 lbs.

3. If the pressure at one edge of a wall is zero, and the wall is 2 ft. 3 ins. thick, find the weight per lineal foot that can be safely placed on it, and the point of application.

Answer, 8·1 tons at 9 ins. from the other edge.

4. The water is level with the top of a masonry wall which is 6 ft. thick; find the maximum height of the wall consistent with safety, and the corresponding maximum pressure on the masonry.

Answer, 9 ft. 0 in.; 17½ lbs. per square inch.

5. A wrought iron tank is 40 ft. diameter, and 20 ft. deep; find the strain on the metal at the bottom per inch in height, when it is full of oil. *Answer*, $\frac{5}{6}$ ton.

6. In the last question give the number of $\frac{5}{8}$ in. rivets required per lineal foot, and the necessary thickness of the iron, the rivet holes being punched and the resistance of the metal to tension and shearing being taken as 5 tons per square inch. *Answer*, 7 rivets; thickness, ·30 in. or $\frac{5}{16}$ in.

7. In the last question, give the number of rivets and thickness of plate at 10 ft. deep.
Answer, 4 rivets; thickness, ·11 in. or $\frac{7}{64}$ in.

8. In question 5, give the strain if the tank is full of salt water. *Answer*, ·95 ton.

9. A cast iron pipe 4 ins. diameter has to be tested with a head of 300 ft. of water, the resistance of the metal being taken at $3\frac{1}{2}$ tons per square inch, find the thickness.
Answer, ·033 in.

10. Find the pressure in lbs. per square inch which will burst a 3 in. cast iron pipe, if the thickness of the metal is $\frac{1}{4}$ in. *Answer*, 2613 lbs.

11. The plating of a locomotive boiler 4 ft. diameter is $\frac{1}{2}$ in. thick; find the pressure of steam that it will sustain when s is put equal to 5 tons, and the rivets are $\frac{3}{4}$ in. diameter, and $1\frac{3}{4}$ ins. pitch.
Answer, 116 lbs. per square inch.

12. An hydraulic press cylinder is 10 ins. internal, and 20 ins. external, diameter. If the metal on the inside is strained to the extent of 6 tons per square inch, to what extent will that on the outside be strained and what pressure will the pipe sustain?
Answer, $1\frac{1}{2}$ tons, and 3 tons per square inch.

13. Two plates of wrought iron, 18 ins. wide and $\frac{1}{2}$ in. thick, are connected by two cover plates and $\frac{3}{4}$ in. rivets $4\frac{1}{2}$ ins. pitch; give the required thickness of the cover plates, and the number of rivets necessary.
Answer, $\frac{1}{4}$ in.; 9.

14. How many rivets would be required if the plates were lap jointed? *Answer*, 17.

15. What tensile strain may be safely put on these plates?
Answer, $36\frac{1}{4}$ tons.

D

16. Two girders, 3 ft. deep, carry a single line of railway over a span of 30 ft.; how many $\frac{7}{8}$ in. rivets must be placed at a joint 6 ft. from the abutment, if the web plates are $\frac{3}{8}$ in. thick?

Answer, 3.

17. How many should there be at the abutment?

Answer, 5.

18. A footbridge is 50 ft. span and 6 ft. wide; if the girders are of the lattice type, 5 ft. deep and formed in 10 bays by verticals for the struts and single diagonals for the ties, how many $\frac{5}{8}$ in. rivets would be required for the diagonal next the abutment?

Answer, 4, if in single shear.

19. Two plates, each 20 ins. wide and $\frac{1}{2}$ in. thick, are joined by two $\frac{3}{8}$ in. cover plates by means of 4 rows of 5 $\frac{11}{16}$ in. rivets; find the breaking weight, the tensile resistance of the metal being 20·6 tons per square inch.

Answer, 153 tons to shear the rivets,
165 " " break the plate,
Exp. (Box, p. 17) No. 1, 164 " tore through the rivet holes,
" " " " 2, 153 " sheared the 10 rivets on one side.

20. A force P is applied at the end of a rigid rod, and is resisted by three parallel and equally spaced forces p_1, p_2, p_3, p_1 being directly opposed to P; find their values in terms of P.

Answer, $p_1 = \frac{5}{6}P$, $p_2 = \frac{2}{6}P$, $p_3 = -\frac{1}{6}P$.

21. Draw the lines of resistance in the reservoir wall shown in Fig. 21 when full and when empty, and say whether any part of the masonry is exposed to a tensile strain, what this is, and what is the greatest compressive strain per square inch on the masonry.

Fig. 21.

22. A tank to hold salt water is 20 ft. diameter, and 10 ft. deep; find the strain per inch in height on the metal at 5 ft. below the top of the tank, if the water is 8 ft. deep.

Answer, 160 lbs.

23. In the last question what will the total pressure on the bottom of the tank be?

Answer, 71·8 tons.

24. If the tank in question 22 is of iron, give the pitch of the rivets if they are $\frac{5}{8}$ in. diameter, and also the thickness of the metal at the bottom.

Answer, $6\frac{1}{2}$ in. and ·06 in.

25. Find the weight which must be applied at a leverage of 170 ins. to break the following cast iron shafts:—

Diameter—	2 ins.	$2\frac{1}{2}$ ins.	$3\frac{1}{4}$ ins.	4 ins.
Answer,	·11 tons	·22 tons	·48 tons	·88 tons.
Exp. (Box, p. 298)	·11 ”	·18 ”	·52 ”	·86 ”

26. Prove that in the case of a wrought iron tube, when no part of the metal is strained to a greater extent than 10 tons per square inch,

$$Wl = 5\frac{\pi}{r}\left\{r^4 - r_1{}^4\right\}.$$

27. Find W for a tube of 2 ins. external diameter and $\frac{1}{8}$ in. thick, the leverage being 2 ft. and the maximum strain on the metal 10 tons per square inch.

Answer, 5·4 cwt.

28. The horizontal member of a channel iron has a strain of $1\frac{7}{8}$ tons transmitted to it at one edge; find the maximum unit strain on the metal if it is 3 ins. wide and $\frac{1}{2}$ in. thick.

Answer, 5 tons per square inch.

CHAPTER IV.

TRANSVERSE STRENGTH.

29. *To find the deflection of a beam carrying a weight at the centre.*

Let the beam $B\,B'$ (Fig. 22) carry a weight W at its centre, and be deflected to the extent of D by it, and let l be its span.

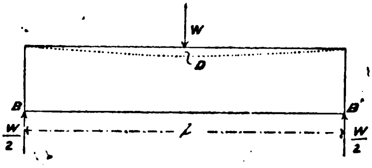


FIG. 22.

The beam will be supported at B and B' by the reactions $\frac{W}{2}$, and in calculating the distance D, which is what is to be found, we may consider half the beam $B\,B'$ as a cantilever acted on by an upward pressure of $\frac{W}{2}$ at a distance of $\frac{l}{2}$; for it amounts to the same thing whether we consider the beam supported at B and B' and deflected downwards at the centre, or supported at the centre and deflected upwards at the ends, as in Fig. 23.*

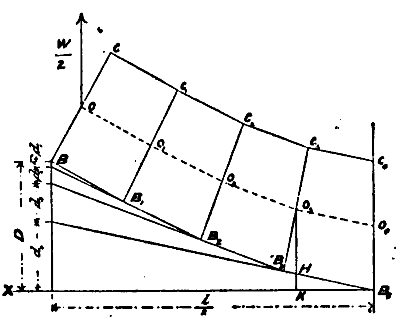


FIG. 23.

* It will be noted that the figure is very much exaggerated as regards D.

Instead of taking an indefinite number n we will, for simplicity, assume this cantilever to be divided by vertical lines into four parts, each equal to a, before it is strained; we shall then have $4a = \frac{l}{2}$, and we will call s_1, s_2, s_3, s_4, the strains at the extreme fibres in each of these, either at the top or the bottom, caused by the reaction $\frac{W}{2}$.

Then we have, by taking moments around the centre B_4, and calling the moment of resistance of the beam, M:—

$$\left.\begin{aligned} s_4\, M &= \frac{W}{2} \cdot 4\, a \\ \text{and so also} \qquad s_3\, M &= \frac{W}{2} \cdot 3\, a \\ s_2\, M &= \frac{W}{2} \cdot 2\, a \\ s_1\, M &= \frac{W}{2} \cdot 1\, a \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

Now, so long as the beam, if of wrought iron, is not strained to a greater extent than 8 or 10 tons per square inch, the top or bottom of each length a will be shortened or lengthened by an amount varying in direct proportion to the strain; so that if the bottom in the portion which sustains a strain of s_1 is lengthened by δa, it will, in that portion which sustains a strain of s_4, be lengthened by $4\ \delta a$.

We will consider the portion $C_3\ B_3\ C_4\ B_4$, in which the strain is s_4, and the bottom $B_3\ B_4$ consequently lengthened from a to $a + 4\ \delta a$, and the top shortened from a to $a - 4\ \delta a$.

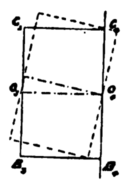


FIG. 24.

The strain, if we consider $B_4\ C_4$ fixed, will cause $C_3\ B_3\ C_4\ B_4$ to revolve about the point O_4 (Fig. 24) until $C_3\ C_4$ becomes

$a-4\,\delta a$ and B_3B_4, $a+4\,\delta a$, and then B_3B_4 will make an angle B_3B_4X, with B_4X, which angle we will call θ_4. From O_3 let fall the perpendicular O_3K on B_4X cutting B_3B_4 in H (Fig. 23). Then, since the angle $B_3HO_3 = KHB_4$ and the angles HB_3O_3, HKB_4 are right angles

$$\text{the angle } B_3O_3H = B_3B_4X \quad = \theta_4,$$

and calling the depth h,

$$\tan\theta_4 = \tan B_3O_3H = \frac{B_3H}{O_3B_3} = 2\frac{4\,\delta a}{h},$$

also
$$\tan\theta_4 = \frac{d_4}{4a},$$

$\therefore$

$$\left.\begin{aligned} d_4 &= 2.4^2.\frac{a\delta a}{h} \\ d_3 &= 2.3^2.\frac{a\delta a}{h} \\ d_2 &= 2.2^2.\frac{a\delta a}{h} \\ d_1 &= 2.1^2.\frac{a\delta a}{h} \end{aligned}\right\} \quad \ldots\ldots \quad (2)$$

so

Hence $D = d_1 + d_2 + d_3 + d_4$.

$$= 2\,\frac{a\delta a}{h}(1^2+2^2+3^2+4^2).$$

If we had divided the beam into n parts, we should have had in the same way

$$D = 2.\frac{a\delta a}{h}\left\{1^2+2^2+\ \ldots\ +n^2\right\},$$

$$= 2.\frac{a\delta a}{h}\frac{n(n+1)(2n+1)}{6}. \quad \ldots\ldots \quad (3)$$

Now the maximum extension $s_nE = n\frac{\delta a}{a}$, E representing the extension or compression of a bar caused by a strain of a ton per square inch, if s_n is in tons, in terms of its length.

$$\text{Hence } D = 2\frac{a^2 s_n}{h} \cdot E \cdot \frac{(n+1)(2n+1)}{6}$$

$$= 2\frac{(an)^2}{3h} s_n E, \text{ where } n \text{ is large for then practically } n+1=n,$$

$$= \frac{l^2}{6h} s_n E, \text{ putting } \frac{l}{2} \text{ for } an \quad . \quad . \quad . \quad . \quad (4)$$

$$= \frac{Wl^3}{24Mh} E, \text{ since } s_n M = \frac{W}{2} \cdot na = \frac{W}{2}\frac{l}{2} \quad . \quad (5)$$

30. *To find the deflection when the beam is rectangular.* This is the simplest case that can occur, but in all cases the principle is the same.

Let $ABCD$ (Fig. 25) be the cross section of the beam in Fig. 22, and let its breadth be b and depth h.

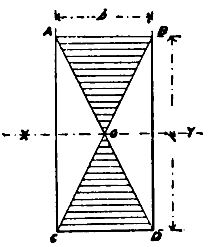


Fig. 25.

If the beam is of wrought iron, and the fibres are not strained to a greater extent than 8 or 10 tons per square inch, the strain will be a compressive strain, decreasing regularly from a maximum at AB to zero at XY, and a tensile strain increasing from zero at XY to a maximum at CD.

If therefore the strain at AB is s_n, it will be zero at XY and $\frac{s_n}{2}$ midway between, and the total strain may therefore be taken as $s_n \times$ (area of the triangle ABO) acting at its centre of gravity, for it is evident that this coincides with the centre of resistance.

Hence the moment of resistance $s_n M$ of the beam will be

$$s_n \frac{bh}{4} \frac{2}{3} h, \text{ or } s_n \frac{bh^2}{6}.$$

$$\text{Thus (5) becomes } D = \frac{Wl^3}{4bh^3} E \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

31. We have taken the case of a wrought iron beam because up to the limit of 8 or 10 tons per square inch the increase in the length of a bar under a tensile strain is very regular, amounting to ·0001 of its length for each ton per square inch, and the same holds good with the compressive strains.

Thus if $s_{\prime\prime}$ acting at AB is $s_{\prime\prime}$ tons per square inch, the top will be compressed ·0008 of its length, and midway to XY ·0004 of its length, while at CD it will be extended ·0008.

With cast iron, however, the compression and extension are not quite the same, nor is the latter regular for each ton strain. Hence not only is XY not quite in the centre of the beam, but the lines from O to A, B, C, and D are, for strains under about 7 tons per square inch, slightly curved.

32. *To find the deflection of a wrought iron bar* 2 *ins. square, carrying a ton at the centre of a* 33 *in. span.*

We have $l=33$ ins., $W=1$ ton, $E=\cdot 0001$, $b=2$ ins., and $h=2$ ins.,

$$\therefore \quad D = \frac{33^3.1}{4.2.2^3} \times \cdot 0001,$$

$$= \cdot 056.$$

For a bar 3 ins. deep and $1\frac{1}{2}$ ins. wide,

$$D = \frac{2^3}{3^3} \cdot \frac{2}{1\frac{1}{2}} \times \cdot 056,$$

$$= \cdot 022.$$

For a bar $2\frac{1}{2}$ ins. deep and $1\frac{1}{2}$ ins. wide,

$$D = \frac{2^3}{(2\frac{1}{2})^3} \cdot \frac{2}{1\frac{1}{2}} \times \cdot 056,$$

$$= \cdot 038.$$

These three results agree exactly with the experiments given on page 104 of Baker's "Beams, Columns, and Arches."

33. *To find the deflection of a beam of fir* 2 *ins. deep,* $1\frac{1}{2}$ *ins. wide and* 3 *ft. span, with a weight of* 120 *lbs. at its centre on the assumption that E is* ·0016.

$$\text{We have } D = \frac{36^3 \frac{120}{2240}}{4.1\frac{1}{2}.2^3} \times \cdot 0016,$$

$$= \cdot 0833.$$

The experiment (Barlow, p. 47) gave ·09.

34. *To find the deflection of a deal beam* 60 *ins. span,* $2\frac{5}{8}$ *ins. wide and* $3\frac{1}{8}$ *ins. deep, with* 6 *cwt. at its centre, the value of E being taken as* ·003.

$$\text{We have } D = \frac{60^3 \frac{6}{20}}{4.2\frac{5}{8}.(3\frac{1}{8})^3} \times \cdot 303,$$

$$= \cdot 6.$$

The author's experiment gave $\frac{1}{2}$ in.

35. *To find the breaking weight of a rectangular beam.*

We will again take the case of a wrought iron beam, and assume the resistance to tension and compression to be 22 tons per square inch. If the beam is just on the point of breaking the extreme fibres will be strained to the extent of 22 tons, which will cause them to be lengthened or depressed ·034 of their length. (Baker, p. 102.)

Thus those half way to the neutral axis will be lengthened ·017, which would be produced by a strain of 18 tons and which shows that this strain is there acting.

So also those fibres three parts of the way to the neutral axis will be lengthened ·0085, which indicates a strain of 16 tons.

By plotting these strains on the cross section of the beam (Fig. 26) it will be seen that instead of

$$M=\frac{b}{2}\cdot\frac{h}{2}\cdot\frac{2}{3}h=\frac{bh^2}{6}$$

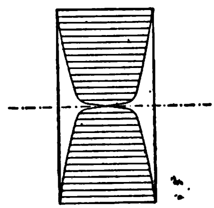


FIG. 26.

we have very nearly

$$M=\frac{bh}{2}\cdot\frac{h}{2}\quad=\frac{bh^2}{4}.$$

Hence $\frac{W}{2}\cdot\frac{l}{2}=s'M\quad=s'\frac{bh^2}{4}$,

or $W=s'\cdot\frac{bh^2}{l}$ where W is in tons and l in ins.,

$=\frac{s'}{12}\cdot\frac{bh^2}{l}$ where W is in tons and l in ft.

The resistance of wrought iron to compression is rather less than to tension, so the value of s' should be put somewhat under 22 tons, and, as the fibres near the neutral axis do not give out the full resistance, 18 may be taken as a fair average value for it.

Thus we have, where W is in tons and l in feet,

$$W=\frac{3}{2}\cdot\frac{bh^2}{l}.$$

36. In calculating the load that can be put on to the beam without straining it beyond the limit of elasticity, we must take the formula

$M=\frac{bh^2}{6}$ as obtained from Fig. 25,

and not $=\frac{bh^2}{4}$ " " Fig. 26.

37. *To find the load that the beam will carry without being strained beyond the limit of elasticity at any point.*

We have $\frac{W}{2}\cdot\frac{l}{2}=sM,$

or $W=s.\frac{4M}{l},$

$=10.\frac{4bh^2}{6l},$ in the case of a rectangular beam, in which s is taken as 10 tons,

$=\frac{2}{3}.10.\frac{bh^2}{l},$ where W is in tons, and b, d, and l are in inches.

38. For timber we have no direct experiments to give the value of E. This has, however, been obtained from experiments on the deflection of beams on the assumption that it is the same for compressive and tensile strains as given in Barlow's "Strength of Materials," page 82. As the pieces tested were 7 ft. long and 2 ins. square, the formula

$$D=\frac{Wl^3}{4bh^3}E \text{ becomes } D=\frac{84^3}{4.2.2^3}W.E,$$

$$i.e.\ E=\frac{D}{21^3W}=\frac{D}{W}\times\cdot 24, \text{ where } W \text{ is in lbs.}$$

Thus $E=\frac{1\cdot 15}{300}\times\cdot 24=\cdot 0009$ for Teak,

$=\frac{1\cdot 60}{150}\times\cdot 24=\cdot 0025$ „ Oak, English,

$=\frac{1\cdot 28}{200}\times\cdot 24=\cdot 0015$ „ „ „

$=\frac{1\cdot 10}{225}\times\cdot 24=\cdot 0011$ „ „ Canadian,

$=\frac{1\cdot 60}{200}\times\cdot 24=\cdot 0019$ „ „ Dantzic,

$=\frac{1\cdot 40}{150}\times\cdot 24=\cdot 0022$ „ „ Adriatic,

$=\frac{1\cdot 25}{225}\times\cdot 24=\cdot 0013$ „ Ash,

$$\text{Thus } E=\frac{1\cdot00}{150}\times\cdot24=\cdot0016 \text{ for Beech,}$$
$$=\frac{1\cdot60}{125}\times\cdot24=\cdot0030 \text{ ,, Elm,}$$
$$=\frac{1\cdot14}{150}\times\cdot24=\cdot0018 \text{ ,, Pine, Pitch,}$$
$$=\frac{0\cdot75}{150}\times\cdot24=\cdot0012 \text{ ,, ,, Red,}$$
$$=\frac{0\cdot93}{150}\times\cdot24=\cdot0015 \text{ ,, Fir, New England,}$$
$$=\frac{0\cdot90}{125}\times\cdot24=\cdot0017 \text{ ,, ,, Riga,}$$
$$=\frac{1\cdot40}{125}\times\cdot24=\cdot0027 \text{ ,, ,, Mar Forest,}$$
$$=\frac{1\cdot90}{125}\times\cdot24=\cdot0036 \text{ ,, Larch.}$$

39. The deflection is constant up to nearly half the breaking weight, and then becomes irregular. We may therefore put $\frac{1}{3}$rd the breaking weight on timber with safety, but except in the most temporary structures it is better not to put more than $\frac{1}{5}$th or $\frac{1}{6}$th on it.

40. As we have no experiments giving the value of E for each ton load per square inch in tension and compression, it is difficult to establish a theory for the breaking weight as has been done in the case of wrought iron. The experiments just alluded to, however, gave the breaking weights for the different pieces, and the position of the neutral axis was found by careful observation. In some it was 1·3 ins. from the top of the beam, and in others 1·2 ins., which is where we should have expected it to be, as the tensile strength of most of the woods is about double the compressive strength, so that the beam on the point of breaking exposes $\frac{2}{3}$rds of its section to resist the crushing and $\frac{1}{3}$rd to resist the tensile strains. When first strained

the neutral axis would be at the centre of the beam, on the assumption that E is the same for tension and compression, but before the breaking point was reached it would move lower down as shown in Fig. 27.

If the fibres are extended as in the case of wrought iron, at the breaking point, we should have

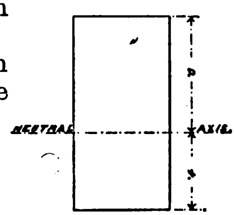


FIG. 27.

$$\frac{W}{2} \cdot \frac{l}{2} = s' M = s'b \,.\, \cdot 4h \, \frac{h}{2} \text{ nearly,}$$
$$= s'bh^2 \,.\, \cdot 2,$$

where s' is the resistance to the tensile strain.

Or if the elasticity is perfect right up to the breaking point,

$$sM = s.\tfrac{1}{2}\,(b \times \cdot 4h)\,\tfrac{2}{3}h,$$
$$= sbh^2 \times \cdot 133.$$

The true value is probably something between these, or say:—

$$\frac{W}{2} \cdot \frac{l}{2} = sbh^2 \times \cdot 175,$$

or $\quad W = \cdot 7 \dfrac{bh^2}{l} s,$

which in the case of the 2 ins. × 2 ins. pieces becomes

$$W = \cdot 7 \frac{2^3}{84} s = \frac{s}{15} \text{ where } W \text{ and } s \text{ are in tons,}$$

so, $\quad s = \dfrac{W}{150} \qquad$,, W is in lbs. and s in tons.

or $\quad = \dfrac{W}{131}$ in the case of Ash, Red Pine, and Fir;

for with these woods the neutral axis was 1·3 from the top instead of 1·2

and so $s = \dfrac{8}{7} \cdot \dfrac{W}{150}.$

The following table gives the value of W for the different materials tested and the consequent mean or approximate value of s:—

	W in lbs.	s in tons.
Teak	820 to 1020	6
Oak, English	421 "	3
" Canadian	650 " 708	$4\frac{1}{2}$
" Dantzic	520 " 580	$3\frac{1}{2}$
Beech	565 " 615	4
Elm	368 " 398	$2\frac{1}{2}$
Pitch Pine	595 " 650	4
Larch	300 " 340	2
Ash	760 " 780	6
Red Pine	470 " 530	4
Fir, New England	403 " 446	3
" Riga	406 " 440	3
" Mar Forest	360 " 465	3

Direct Experiment gave $6\frac{1}{2}$ tons for Teak, 4 to 5 tons for English Oak, and 5 tons for Beech.

In calculating the strength of the foregoing we may take as the value of s, for the breaking weights and safe loads,

	Tons.		Ton.	
For Teak and Ash	6	and	1	respectively.
" Oak and Pitch Pine	4	"	$\frac{2}{3}$	"
" Elm, Larch, and Fir	$2\frac{1}{2}$	"	$\frac{1}{2}$	"

in the formula

$$\frac{W}{2} \cdot \frac{l}{2} = sM = s \cdot \frac{bh^2}{6}, \text{ or } W = \frac{2}{3} \cdot \frac{bh^2}{l} s,$$

where W and s are in tons, and b, h and l in inches.

It will be noticed that if $\frac{1}{6}$th the breaking weight is put on a beam, the extreme fibres will be strained to a *greater* extent than $\frac{1}{6}$th the breaking tensile or compressive weight, for the moment of resistance will be as in Fig. 26 in the one case, and as in Fig. 25 in the other.

41. With cast iron, owing to the unequal extension under a tensile strain, the deflection is irregular. The extension varies from ·00016 of the length per ton per square inch under a strain of one ton, to ·00024 per ton per square inch under a strain of 7 tons. The compression within these limits is about ·00018, and is nearly constant.

Thus for a beam in which the extreme fibres are not strained to a greater extent than 5 tons per square inch, the deflection should practically follow the same rule as in the case of wrought iron, the neutral axis being about the centre of the beam.

42. *To find the deflection of a cast iron bar* 1 *in. square with a load of* 1 *lb. at the centre of a* 12 *in. span.*

We have $$D=\frac{Wl^3}{4bh^3}E,$$

Now since $$\frac{W}{2}\cdot\frac{l}{2}=sM \quad =s\cdot\frac{bh^2}{6},$$

$$i.e.\quad s=\frac{3}{2}\frac{Wl}{bh^2}=\frac{3}{2}\frac{\frac{1}{2240}\,.\,12}{1\,.\,1^2}=\frac{1}{125}\text{ ton},$$

E may be put at say ·00015, and we get

$$D=\frac{\frac{1}{2240}\,.\,12^3}{4\times1\times1^3}\cdot{\cdot}00015,$$
$$={\cdot}000029,$$

and this is the mean result of a large number of experiments. (Box, p. 186.)

43. *To find the deflection of a cast iron bar* ·65 *ins. deep and* 1·3 *ins. wide with a load of* 120 *lbs. at the centre of a* 35 *in. span.*

Since $$s=\frac{3}{2}\frac{Wl}{bh^2}=\frac{3}{2}\cdot\frac{\frac{120}{2240}\times35}{1{\cdot}3\times({\cdot}65)^2},$$
$$=5\text{ tons},$$

We may put $E={\cdot}00018.$

Hence $$D=\frac{\frac{120}{2240}\times35^3}{4\times1{\cdot}3\times({\cdot}65)^3}\,.\,{\cdot}00018,$$
$$={\cdot}29.$$

Tested with a weight of 162 lbs. the deflection was ·27. (Baker, 100.)

44. For the breaking weight of cast iron:—

Since the tensile resistance is $\frac{1}{6}$th the compressive, we might expect to find the neutral axis dividing the beam in the proportion of 6 to 1, as in the case of timber, in which the tensile resistance is double the compressive, and the neutral axis has been proved by experiment to be at a depth of $\frac{2}{3}$rds of the beam.

It seems that until the strain comes to nearly 7 tons per square inch, the neutral axis is nearly at the centre of the beam; whence it follows that the extreme fibres are strained almost up to their breaking weight long before the beam actually does break.

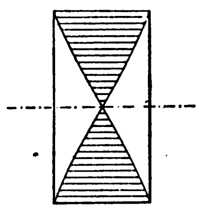


Fig. 28.

We may suppose the beam to be strained as in Fig. 28, when the strain on the extreme fibres is a little under 7 tons.

Any addition to the weight after this will cause the neutral axis to move upwards, until it reaches the position shown in Fig. 29.

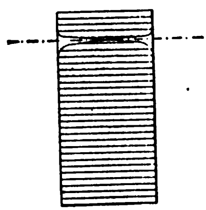


Fig. 29.

If at the breaking point the whole of the top part gave a resistance of 42 tons per square inch, and the whole of the bottom 7 tons, we should have

$$\frac{W}{2} \cdot \frac{l}{2} = 7 \times b . \frac{6}{7} h \frac{h}{2} \quad \text{or} \quad = 42\, b . \frac{h}{7} . \frac{h}{2},$$

$$= 3bh^2, \qquad \text{,,} \qquad = 3bh^2,$$

$$\textit{i.e. } W = 12\,\frac{bh^2}{l}, \text{ where } l \text{ is in inches,}$$

$$= \frac{bh^2}{l} \qquad \text{,,} \quad \text{,,} \qquad \text{feet.}$$

By actual experiment, as we should expect, W is not quite $\frac{bh^2}{l}$, but the mean of a large number of experiments has given

$$W = \cdot 9\,\frac{bh^2}{l},$$

and this may be taken as the correct formula for calculating solid beams of cast iron.

It must be noted, however, that if we take the safe load as $\frac{1}{3}$rd of the breaking weight, we shall put a strain on the extreme fibres exposed to tension of much more than $\frac{7}{3}$ or $2\frac{1}{3}$ tons, for we have, as in Fig. 28,

$$\frac{W'}{2}\cdot\frac{l}{2} = s'\frac{bh^2}{6},$$

or
$$W' = \frac{s'}{18}\cdot\frac{bh^2}{l},$$

and so, if $W' = \frac{W}{3}$, we have $\frac{s'}{18}\cdot\frac{bh^2}{l} = \frac{1}{3} \times \cdot 9\frac{bh^2}{l}$,

whence we get $s' = 18 \times \cdot 3 = 5\cdot 4$ tons.

45. *To find the breaking weight of a bar of cast iron* 2 *ins. square and* 20 *ins. span, the resistance of the metal to tension being* 12 *tons per square inch.*

The resistance of the metal to compression is not stated, so we will take it at 42 tons per square inch at the point of fracture.

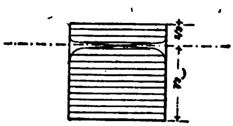


Fig. 30.

The neutral axis will then divide the bar in the proportion of 42 : 12, *i.e.* of 7 : 2, and we have

$$\frac{W}{2}\cdot\frac{l}{2} = 2\text{ ins.} \times \frac{7}{9}\,2\text{ ins.} \times 12\text{ tons} \times 1\text{ in.} \times \cdot 9,$$

from which we get $W = 6\cdot 72$ tons.

Experiment gave from 5·8 to 6·7 tons (Barlow, p. 166).

46. *To find the deflection of a wrought* **T. I.** *bar* 3 *ins. by* 3 *ins. by* ½ *in.* 25 *ins. span, when the extreme fibres are strained to* 10 *tons per square inch; also the central load that it will carry when so strained.*

The strain of 10 tons per square inch at the extreme fibres will be reduced to zero at the neutral axis.

Hence the resistance of the top portion (Fig. 31) will be equal to the area of the shaded part multiplied by 10 tons.

Now, since the extension and compression of wrought iron are the same per ton within the elastic limits, the web will be lengthened at the depth of ·93 below the neutral axis to the same extent as it is compressed at ·93 above it, and the extreme fibres in the flange, instead of being strained to the extent of 10 tons per square inch, as they would be in a beam of symmetrical cross section, are only strained to $\frac{\cdot 93}{2\cdot 07} \times 10$, or to 4·5 tons.

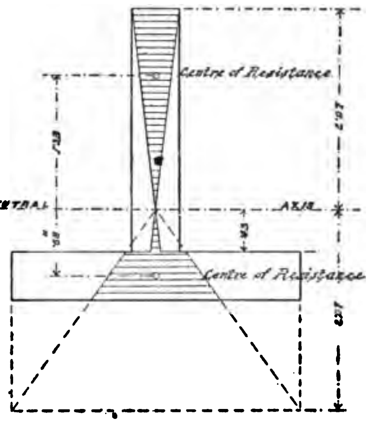


Fig. 31.

The position of the neutral axis is a little difficult to find. It must be in such a position that the area of the shaded part above it is equal to that below it; that is to say, the resistance offered to compression above the axis must equal the resistance to tension below, as in all girders. The simplest way to find it is to assume some position and then draw the lines as

in Fig. 31, and then if the top area is larger than the bottom, to try another position nearer the top, and so on again until the areas are equal or nearly so.

It will of course be noted that the neutral axis passes through the centre of gravity of the beam, so that this is really what has to be found.*

In Fig. 31, it has by the above process been found to be at a depth of 2·07 from the top, for we have:—

$$\tfrac{1}{2} \times 2{\cdot}07 \text{ ins.} \times \tfrac{1}{2} \text{ in.} \qquad = {\cdot}517,$$

$$\text{and } \tfrac{1}{2} \times {\cdot}43 \times \frac{{\cdot}43}{2{\cdot}07} \times {\cdot}50 + {\cdot}50 \times \frac{{\cdot}68}{2{\cdot}07} \times 3 = {\cdot}515.$$

To find the effective depth of the beam it is necessary to find the centre of resistance, or, which is the same thing, the centre of gravity of each of the shaded parts.

This for the top is $\frac{2}{3}$rds of 2·07 $= 1{\cdot}38$ from the neutral axis, and for the bottom

$$\frac{\left(\frac{1}{2}\,\frac{{\cdot}43}{2{\cdot}07} \times {\cdot}5 \times {\cdot}43\right)\frac{2}{3} \times {\cdot}43}{{\cdot}515} + \frac{\left(\frac{{\cdot}43}{2{\cdot}07} \times 3 \times {\cdot}50\right) \times {\cdot}68}{{\cdot}515} + \frac{\left(\frac{1}{2}\,\frac{{\cdot}50}{2{\cdot}07} \times 3 \times {\cdot}50\right) \times {\cdot}763}{{\cdot}515},$$

$$i.e.\ \frac{{\cdot}0223 \times \frac{2}{3} \times {\cdot}43 + {\cdot}3115 \times {\cdot}68 + {\cdot}1812 \times {\cdot}763}{{\cdot}515}, \text{ or } \frac{356{\cdot}5}{515}$$

i.e. ·69.

---

* In the simple case under consideration the distance of the centre of gravity from the top of the T. I. is $\dfrac{1\frac{1}{2} \times 2\frac{3}{4} + 1\frac{1}{4} \times 1\frac{1}{4}}{1\frac{1}{2} + 1\frac{1}{4}}$. It is only for the more complicated sections that the method given in the text of finding it is applicable.

To find the deflection we have

$$D=\frac{l^2 s_n E}{6h}=\frac{25^2.10.\cdot0001}{6\times 4\cdot 14}$$
$$=\cdot 025.$$

And to find the weight that can be carried at the centre, we have

$$\frac{W}{2}\cdot\frac{l}{2}=M.10=\cdot 515\times(\cdot 69+1\cdot 38)\times 10,$$
$$=10\cdot 66.$$

Hence $$W=\frac{4}{25}\times 10\cdot 66,$$
$$=1\cdot 7 \text{ tons}.$$

Thus a weight of 1·7 tons will deflect the T. I. ·025 ins., and induce a strain of 10 tons per square inch on the extreme fibres.

47. The symbol M has been put for the moment of resistance of a beam, and has been obtained graphically in the case of a rectangular bar, such as $ABCD$ (Fig. 32), by drawing lines from the points A, B, C, D to O on the neutral axis. The area of one of the triangles AOB, COD multiplied by the distance apart of their centres of gravity, or, which amounts to the same thing, the area of the two triangles each multiplied by the distance of its centre of gravity from the neutral axis, gives the value of M as already explained. Thus we have from the figure

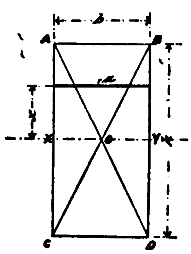


FIG. 32.

$$M=\text{area of } AOB\times\frac{2}{3}BD=\frac{1}{2}\cdot\frac{bh}{2}\frac{2h}{3}=\frac{bh^2}{6},$$

$$\text{or } M=\text{area of } AOB\times\frac{2}{3}BY+\text{area of } COD\times\frac{2}{3}DY$$
$$=\frac{1}{2}\cdot\frac{bh}{2}\left\{\frac{2}{3}\cdot\frac{h}{2}+\frac{2}{3}\cdot\frac{h}{2}\right\}=\frac{bh^2}{6}.$$

This does not mean that only the shaded portions of the beam are strained, but that the unit strain decreases from a maximum at AB to zero at the neutral axis XY. Thus if we take a fibre distant x from XY, the unit strain on it will be $\frac{x}{\frac{h}{2}}s$, s being the maximum unit strain.

We may express this algebraically thus:—

Let the whole area of the cross section $ABCD$ be divided into a number of small areas equal to a_1, a_2, a_3, . . . a_n distant x_1, x_2, x_3, . . . x_n respectively from the neutral axis, and we shall have

$$Ms = a_1x_1\frac{x_1}{\frac{h}{2}}s + a_2x_2\frac{x_2}{\frac{h}{2}}s + \quad . \quad . \quad . \quad . + a_nx_n\frac{x_n}{\frac{h}{2}}s,$$

$$= \left\{ a_1x_1^2 + a_2x_2^2 \quad + \quad . \quad . \quad . \quad + a_nx_n^2 \right\} \frac{s}{\frac{h}{2}},$$

i.e. $M\frac{h}{2} = a_1x_1^2 \quad + a_2x_2^2 \quad + \quad . \quad . \quad . \quad + a_nx_n^2.$

The symbol Σax^2 may be put for the sum

$$a_1\,x_1^2 + a_2\,x_2^2 + \quad . \quad . \quad . \quad + a_nx_n^2,$$

Thus we have $M\frac{h}{2} = \Sigma ax^2$.

I is generally put for Σax^2. It is called the *Moment of Inertia* of the cross section, and is, as we have seen, the sum of the small areas, into which this is divided, each multiplied by the square of its distance from the neutral axis.

Again, if A is the whole area, Ar^2 may be put for I, r being called the "*Radius of Gyration.*"

Thus we have $M\frac{h}{2} = \Sigma ax^2 = I = Ar^2$.

48. We will conclude this chapter with a summary of the results obtained.

(*a*) To find the deflection of a beam of uniform section, we have the formula

$$D = \frac{Wl^3}{24Mh}E,$$

$$= \frac{Wl^3}{4bh^3}E \text{ for a rectangular beam.}$$

In this D is the deflection in inches at the centre,
- W „ weight „ tons „
- l „ span „ inches,
- M „ moment of resistance of the cross section in inches,
- b „ breadth of the beam in inches,
- h „ depth of the beam in inches or rather twice the distance of the fibres most strained from the neutral axis,
- E „ the part of its length that a strain of a ton per square inch would lengthen each material.

Except in the case of cast iron the value of E is constant up to nearly half the breaking weight, after which point it increases rapidly. It may be taken as follows:—

Cast Iron	·00018	Teak	·001
Wrought Iron	·00010	Oak and Pitch Pine	·002
Steel	·00008	Fir	·003

(*b*) To find the deflection when the moment of resistance is proportional to the strain, we have from example 53 *seq.*

$$D = \frac{Wl^3}{16Mh}E.$$

(*c*) The strain on the extreme fibres is given by the formula

$$s = \frac{Wl}{4M}, \text{ where } s \text{ is in tons per square inch.}$$

For wrought iron and steel this is the formula from which the strength of a beam may be calculated, and in order that no portion of the material may be strained beyond the elastic limit, s should not exceed 10 tons for wrought iron, or about 20 tons for steel; but if the ultimate resistance s' is exactly double s, the breaking weight W' will be *more* than double W, as will be seen from (35), where we have for the breaking weight

$$W' = 4\frac{M}{l}s',$$
$$= \frac{bh^2}{l} \times \cdot 9s \text{ for a rectangular beam,}$$

the value of s' being for wrought iron 22 tons, and for steel 36 tons.

(*d*) With timber we have for the breaking weight

$$W = \frac{bh^2}{l} \times \cdot 7s \text{ in some cases, and } \frac{bh^2}{l} \times \cdot 6s \text{ in others,}$$
$$= \frac{bh^2}{l}c,$$

in which the approximate value of c is as follows:—

Teak . .	4	Riga Fir .	2
Oak . .	$2\frac{1}{2}$	Deal . .	2
,, African .	$4\frac{1}{2}$	Ash . .	$3\frac{1}{2}$
Pitch Pine .	3	Elm . .	2
Yellow ,, .	$2\frac{1}{2}$	Mahogany.	3

The deflection is constant up to about half the breaking weight, so that $\frac{1}{3}$rd of this may be put on without injuring the timber. In permanent structures, however, $\frac{1}{5}$th or $\frac{1}{6}$th of the breaking weight should not be exceeded, so that c should be put $\frac{2}{3}$ for Teak, $\frac{1}{2}$ for Oak, Pitch Pine and Ash, and $\frac{1}{3}$ for Fir and Deal.

(*e*) Cast iron is a material which requires quite distinct consideration, owing to the great difference between its resistance to tension and compression. If the tensile resistance of the metal is 7 tons per square inch, and

a beam is so loaded that the strain on the extreme fibres is a little less than 7 tons per square inch, the neutral axis will occupy nearly the same position as it would for a wrought iron beam; but on a further load being added, it will move until at the breaking point, in the case of a rectangular bar, it divides the parts exposed to tension and compression nearly in the proportion of 6 : 1.

In the absence of experiments giving the resistance of the same metal to tension, compression and transverse strain, it is impossible to frame a correct theory; but Examples 40 to 45, 50, 51, and 56 to 64, *seq.*, may be taken as a good guide for calculating the strengths of beams of various sections.

Round bars are 10 per cent. weaker than square bars of the same sectional area, and square bars placed with one of their diagonals vertical 20 per cent. weaker.

For rectangular bars we have the formula

$$W=\frac{bh^2}{l}\times \cdot 9, \text{ where } l \text{ is in feet,}$$

and so for round bars, $W=\frac{bh^2}{l}\times \cdot 8$,

and for rectangular bars placed diagonally,

$$W=\frac{bh^2}{l}\times \cdot 7.$$

In this instance h is the depth of the bar, and not twice the distance of the extreme fibres from the neutral axis as before.

EXAMPLES.

1. Find the deflection of a wrought iron bar 2 ins. square 25 ins. span with a load of 3 tons at the centre.

Answer, ·073 in.

Exp. (Baker, 104), ·079 „

2. What is the greatest load that this bar will carry, if the extreme fibres are not strained to a greater extent than 10 tons per square inch? *Answer*, 2·13 tons.

3. Find the central breaking weight of this bar.
Answer, 5·76 tons.
Exp. (Baker, 31), 6·25 „

4. How may the difference between the calculated and experimental result in (1) be explained?

5. A bar of steel $1\frac{3}{4}$ ins. square sustains a load of 1,000 lbs. at the centre of a 25 in. span; find the deflection.
Answer, ·0015.
Exp. (Baker, 113), ·0015.

6. A plate of steel 18 ins. wide and $\frac{1}{2}$ in. thick sustains a weight of a ton at the centre of a 35 in. span; find the deflection. *Answer*, ·38 in.
Exp. (Baker, 113), ·37 „

7. Find the breaking weight of a bar of steel 1 in. square and 12 ins. long from first principles. *Answer*, 2·7 tons.

8. Find the breaking weight of a steel bar $1\frac{3}{4}$ ins. square and 25 ins. long. *Answer*, 6·9 tons.
Exp. (Baker, 56), 7·3 „

9. How much will a weight of 180 lbs. deflect a fir beam 2 ins. wide, $1\frac{1}{2}$ ins. deep and 6 ft. span, if $E = \cdot 0018$?
Answer, 2·00 ins.
Exp. (Barlow, 47), 1·85 to 2·00 „

10. What will the deflection be if the beam is 3 ins. deep and $1\frac{1}{2}$ in. wide? *Answer*, ·333 in.
Exp. (Barlow, 50), ·375 and ·287 „

11. In the last case give the strain on the extreme fibres.
Answer, ·64 ton.

12. Find the weight which, placed at the centre of a 5 ft. span, will break a piece of deal $2\frac{5}{8}$ ins. wide and $3\frac{1}{8}$ ins. deep, from section 40. *Answer*, ·71 ton.
Exp. (Author), ·81 „

13. Find the central breaking weight of a piece of fir 7 ft. span and 2 ins. square, by section 40. *Answer*, 357 lbs.
Exp. (Barlow, 84), 406 to 440 „

14. Find the deflection of this piece under a central load of 125 lbs., if $E = \cdot 0018$. *Answer*, ·93 in.
Exp., ·94 to ·81 „

15. In the latter case to what extent will the extreme fibres be strained? *Answer*, ·88 ton.

16. Find the deflection of a beam $1\frac{1}{2}$ ins. wide, 3 ins.

deep and 8 ft. span, under a central load of 180 lbs., if $E = \cdot 0018$. *Answer*, ·79 in.

17. What is the maximum induced strain per square inch? *Answer*, 17·14 cwt.

18. The span being 10 feet in (16), give the deflection. *Answer*, 1·54 ins.

19. Give the maximum induced strain in this case. *Answer*, 1·07 tons.

20. Find the strain on the extreme fibres of a beam $1\frac{1}{2}$ ins. deep and 3 ins. wide, when loaded with 80 lbs. at the centre of an 8 ft. span. *Answer*, ·76 ton.

21. Calculate the strength of a beam of oak, from first principles, on the assumption that the neutral axis is at the centre of the beam, the resistance to tension and compression $4\frac{1}{2}$ tons per square inch, and the elasticity perfect to the breaking point. *Answer*, $W = 3\frac{bh^2}{l}$.

22. Find the breaking weight of a beam of oak $4\frac{1}{4}$ ins. square at the centre of a 7 ft. 6 in. span from this.
Answer, 2·56 tons.
Exp. (Barlow, 55), 2·50 „

23. What is the breaking weight if the beam is $5\frac{1}{3}$ ins. square and the span 15 ft.? *Answer*, 2·5 tons.
Exp. (Barlow, 55), 2·5 „

24. What would the ultimate deflection in (22) be, if the law of elasticity held good up to the breaking point?
Answer, 2·8 ins., if $E = \cdot 002$.
Exp. (Barlow, 55), 3·47 „

25. Find the breaking weight of a piece of oak $7\frac{1}{2}$ ins. square and 15 ft. long, by (21). *Answer*, 7 tons.
Exp. (Barlow, 56), 6·5 „

26. Find the breaking weight of a piece of pitch pine 2 ins. square and 10 ft. long, by section 40.
Answer, 3·55 cwt.

27. Find the deflection of a piece of pine 14 ins. wide and 7 ins. deep, with a load of $8\frac{1}{2}$ tons at the centre of a 5 ft. bearing, putting $E = \cdot 003$. *Answer*, ·287 in.
Exp. (Baker, 130), $\begin{cases} \cdot 263 \text{ „} \\ \cdot 425 \text{ „} \end{cases}$

28. In the last example find the value of *s*.
Answer, 1·12 tons.

29. Find the strain on the extreme fibres and the deflection of a cast iron bar 1·3 ins. wide and ·65 in. deep, with a load of 162 lbs. at the centre of a 35 in. span.

Answer, $s = 6·9$ tons,
$D =$ ·39 in.
Exp. (Baker, 100), ·27 „

30. Find the strain on the extreme fibres and the deflection of a wrought iron bar $1\frac{1}{2}$ ins. wide and 3 ins. deep, with a ton at the centre of a 33 in. span. *Answer*, $s = 3·66$ tons,
$D =$ ·022 in.
Exp. (Baker, 104), $\begin{cases} ·022 \text{ „} \\ ·026 \text{ „} \end{cases}$

31. The moment of resistance of a 60 lb. double-headed rail, $4\frac{1}{2}$ ins. deep, is 6·7; find the deflection under a load of a ton at the centre of a 33 in. bearing. *Answer*, ·005 in.
Mean of 33 *Exps.* (Baker, 105), ·005 „

32. Find s and D for a $\frac{1}{2}$ in. steel plate 24 ins. wide, with a ton at the centre of a 75 in. span. *Answer*, $s =$ 18·75.
$D =$ 2·8 ins.
Exp. (Baker, 113), 3·0 „

33. Find the deflection of the 84 lb. rail shown in Fig. 33, when loaded with 2,000 lbs. at the centre of a 60 in. span; find also the strain on the extreme fibres, the depth being $4\frac{1}{2}$ ins. *Answer*, $s = 1·5$ tons,
$D =$ ·014 in.
Exp. (Baker, 114), $\begin{cases} ·012 \text{ „} \\ ·021 \text{ „} \end{cases}$

34. What weight will the rail in the last question carry when the greatest strain on the metal is 10 tons per square inch?

Answer, 5·81 tons.

35. What will the distributed weight be in the last question when the span is 6 ft?

Answer, 9·68 tons.

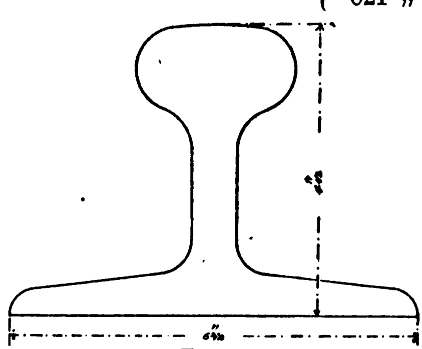


FIG. 33.

36. What is the breaking weight of a bar of cast iron 1 in. square and 54 ins. span? *Answer*, 448 lbs.

Exp. (Baker, 21), { 357 „ to 581 „

37. Find the central breaking weight of a rolled joist 2·56 ins. wide, 3·7 ins. deep, with both web and flanges $\frac{1}{3}$ in. thick, the span being 48 ins. *Answer*, 6·00 tons.

Exp. (Baker, 35), 6·25 „

38. What load can be put on the T. I. in section 46, so that the extreme fibres shall not be strained to a greater extent than 5 tons per square inch, the span being 60 ins.?

Answer, 7·1 cwt.

39. How much will a weight of a pound deflect a bar of wrought iron 1 in. square and 12 in. span?

Answer, ·0000193.

Exp. ·0000192.

40. Find the central breaking weight of a piece of cast T. I. 4 ins. wide, 1·35 ins. deep and $\frac{1}{4}$ in. thick, placed with the web uppermost, the span being 4 ft. 3 ins. and the tensile resistance of the metal $6\frac{1}{2}$ tons per square inch.

Answer, 1,028 lbs.

Exp. (Box, 197), 1,008 „

41. In the last question, what would the weight be if the position of the T. I. was reversed? *Answer*, 253 lbs.

Exp. (Box, 197), 270 „

42. In example 40, if the beam is 5 ins. wide, 1·56 ins. deep, the flange ·3 in. and the web ·365 in. thick, and the span 6 ft. 6 ins., what will the breaking weight be?

Answer, 1198 lbs.

Exp. (Box, 197), 1120 „

43. In the last question what would the weight be if the position of the T. I. was reversed?

Answer, 316 lbs.

Exp. (Box, 197), 364 „

44. Find the deflection of the girder in Fig. 34, the span being

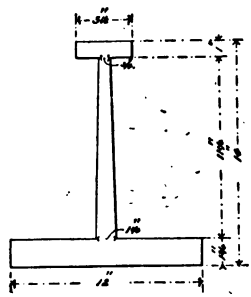


FIG. 34.

16 ft. and the central load 7 tons. *Answer*, 0·25.

Exp. (Box, 202), *mean of* 11 *tests*, 0·24,

(*Reduced from a* 14 *ton load*), „ „ 12 „ 0·26.

45. Find the breaking weight of this girder.

Answer, 38·7 tons.

Mean of 13 *Exps.* (Box, 202), 38·3 „

46. The strain on a piece of **T. I.** $1\frac{1}{4}$ ins. × $1\frac{1}{4}$ ins. × $\frac{1}{4}$ in. is nowhere greater than 10 tons per square inch; find the deflection and load carried if the material is wrought iron and the span 5 ft. *Answer*, Load, 1·2 cwt.

Deflection, ·353 in.

Exp. (Author), ·342 „

47. Find the breaking weight of the **T. I.** in the last example. *Answer*, 4·3 cwt.

Exp. (Author), 4·5 „

48. Find the deflection of a 2 ins. × 2 ins. **T. I.** with 5·8 cwt. at the centre of a 5 ft. span, the vertical web being $\frac{3}{8}$ in. thick, and the horizontal flange $\frac{1}{4}$ in. thick.

Answer, ·29 in.

Exp. (Box, 215), ·26 „

49. Find the breaking weight of this **T. I.** *Answer*, 17·6 cwt.

Exp. (Box, 214), 17·5 „

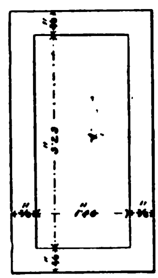


FIG. 35.

50. Four beams, as shown in Fig. 35, were tested by Mr. Clark with a weight at the centre of a 6 ft. span and the breaking weight varied from 2·45 to 2·2 tons, the mean being 2·3. Taking the tensile strength of the metal as 7 tons per square inch, draw approximately on the section the relative strains given out by the different fibres. (Box, 198).

51. Do the same thing in the case of Fig. 36, where three experiments gave 2·00, 2·05 and 2·40 or a mean breaking weight of 2·15 tons.

(Box, 198).

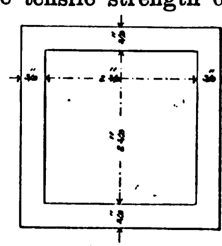


FIG. 36.

52. In example 12 the deflection under a load of 1,100 lbs. was $\frac{13}{16}$; find the value of E.

Answer, ·0025.

53. Find the deflection of a beam in which the moment of resistance is proportional to the strain.

Answer, $D = \frac{Wl^3}{16Mh}E.$

54. Find the deflection of an oak beam 2 ins. square with 196 pounds at the centre of an 8 ft. span, if ·002 is put for E. *Answer*, 2·4 ins.

Mean of a number of Experiments

Riga Fir	(Barlow, 58),		1·3	„
Dantzic Oak	„	„	1·8	„
English „	„	„	1·2	„
Pitch Pine	„	„	1·0	„

55. Find the deflection of a piece of fir 2 ins. deep and 1 in. wide, with a load of 420 pounds at the centre of a 30 in. span.

Answer, if $E = ·003$, ·47 in.

Exp. (Barlow, 73), ·44 „

56. Find the central breaking weight of the cast iron beam in Fig. 37 for a span of 48 ins. *Answer*, ·9 ton.

Mean of 6 *Exps*. (Barlow, 154), 1·1 „

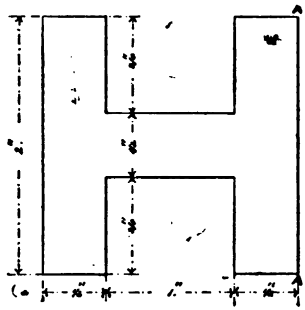


FIG. 37.

57. Find the breaking weight, if the side AA is horizontal. *Answer*, 1·3 tons.

Mean of 4 *Exps*. (Barlow, 154), 1·6 „

58. Find the breaking weight of the cast iron girder shown in Fig. 38, on the assumption that half the web gives out the full strain of 7 tons per square inch, the span being 4 ft. 6 ins.

Answer, 3 tons.

Exp. (Barlow, 173), 3 „

59. Find the breaking weight if the top flange is $1\frac{3}{4}$ ins. by $\frac{1}{4}$ in., and the bottom $1\frac{3}{4}$ ins. by $\frac{1}{2}$ in., the other conditions being the same.

Answer, 3·36 tons.

Exp. (Barlow, 174), 3·30 „

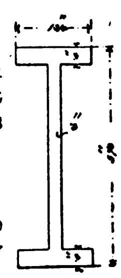


FIG. 38.

60. Find the breaking weight if the top flange is 1 in. by

·3 in., and the bottom 2 in. by ·6 in., the other conditions being the same. *Answer*, 4·0 tons.

Exp. (Barlow, 174), 3·7 ,,

61. Find the breaking weight of the girder in Fig. 39 under the same conditions.

Answer, 4·2 tons.

Exp. (Barlow, 174), 3·9 ,,

62. Find the breaking weight of the girder in Fig. 38 if the top flange is 1 in. by ·34 in. and the bottom 3 ins. by $\frac{1}{2}$ in., the depth and span being as before. *Answer*, 4·9 tons.

Exp. (Barlow, 176), 4·8 ,,

63. Find the breaking weight if the top flange is $1\frac{1}{2}$ ins. by ·3 in., the web ·34 in. thick and the bottom flange $5\frac{1}{8}$ ins. by $\frac{1}{2}$ in., the other conditions being as before.

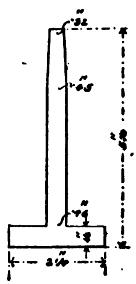


FIG. 39.

Answer, 7·5 tons.

Exp. (Barlow, 177), 7·5 ,,

64. Find the breaking weight of the beam shown in Fig. 40, neglecting the vertical web, the span being 7 feet.

Answer, 5·3 tons.

Exp. (Barlow, 180), 6·1 ,,

65. Give the deflection of a 2 in. square bar of wrought iron with 4,500 lbs. at the centre of a 25 in. span. *Answer*, ·049.

Exp. (Barlow, 256), ·049.

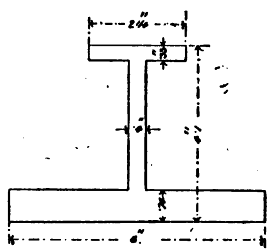


FIG. 40.

66. If the common rafters in a roof are placed 15 ins. apart, centre to centre, and are $2\frac{1}{2}$ ins. wide and $3\frac{1}{4}$ ins. deep, what is the greatest span that should be given to them, the pressure of wind and roof covering being taken at 56 lbs. per square foot vertical? With this span what will the deflection be if E = ·003?

Answer, Span, 6 ft. 10 ins.
Deflection, ·51 in.

67. Find the greatest span and corresponding deflection for 1 in. slate boarding. *Answer*, Span, 5 ft. 2 ins.
Deflection, ·96 in.

68. Find the greatest span and corresponding deflection for 9 in. by 3 in. purlins placed 4 ft. apart and 8 ft. apart.

Answer, Span, 11 ft. 7 ins. and 8 ft. 2 ins.
Deflection, ·53 in. and ·26 in.

69. The joists in the floor of a house are 12 ins. apart centre to centre. If they are $2\frac{1}{2}$ ins. wide and 7 ins. deep, find the maximum span and corresponding deflection.

Answer, Span, 11 ft. 8 ins.
Deflection, ·7 in.

70. The flooring boards of a platform at a passenger station are carried by joists, placed 2 ft. 6 ins. apart centre to centre. The width being 3 ins. and the span 6 ft., find the depth and deflection. *Answer*, Depth, $6\frac{1}{2}$ ins.
Deflection, ·2 in.

71. In the last question, if the platform was for a warehouse, what would the depth and deflection be?

Answer, Depth, 9 ins.
Deflection, ·14 in.

72. The pitch pine cross bearers in a railway bridge are 11 ft. span, and carry a distributed load of 16 tons; give their size and deflection if they are square.

Answer, 13 ins. × 13 ins.
Deflection, ·32 in.

73. If the span in the last question is 16 ft., give the size and deflection. *Answer*, $14\frac{1}{2}$ ins. × $14\frac{1}{2}$ ins.
Deflection, ·64 in.

74. What weight applied at the centre will break a balk of pitch pine 12 ins. square and 20 ft. span?

Answer, $21\frac{1}{2}$ tons.

75. Find the deflection of this timber, when loaded with an equally distributed load of 20 tons. *Answer*, $3\frac{1}{3}$ ins.

76. If a single line of railway is to be carried over an opening by two 12 ins. by 12 ins. balks of Riga fir, what must the maximum span be? *Answer*, 6 ft.

77. What would the span be if the balks were 14 ins. by 14 ins., and of pitch pine? *Answer*, 14 ft. 3 ins.

78. A roadway bridge is formed of fir balks laid close together side by side, and covered with 6 ins. of metalling. If the thickness is 12 ins., find the maximum span.

Answer, 31 ft.

79. In the last case give the proper depth for a span of 12 ft. *Answer*, 4·65 ins.

80. A footbridge floor is 3 ins. thick; find the maximum span. *Answer*, 10 ft. 11 ins.

81. Find the distributed load which a steel joist will carry without any portion of the material being strained to a greater extent than 10 tons per square inch, the span being 20 ft., the width 6 ins., the depth 12 ins. and the metal $\frac{5}{8}$ in. thick. *Answer*, 16·8 tons.

82. Give the deflection of a beam of pitch pine 11·5 ins. wide, 11·4 ins. deep and 12 ft. span, under a central load of (*a*) 9 tons, (*b*) $13\frac{1}{2}$ tons and (*c*) 18 tons.

Answer, ·78, 1·17, 1·56.

Exp. (Min. Inst. C.E., 53–158), ·51, 0·84, 1·37.

83. Give the deflection of a beam of Baltic redwood 11·88 ins. square and 12 ft. span, under a central load of (*a*) $4\frac{1}{2}$ tons, (*b*) 9 tons, (*c*) $13\frac{1}{2}$ tons. *Answer*, ·50, 1·00, 1·50.

Exp. (Min. Inst. C.E., 53–158), ·36, 0·83, 1·40.

84. In examples 82 and 83, give the breaking weights.

Answer, 31·0 tons and 23·3 tons.

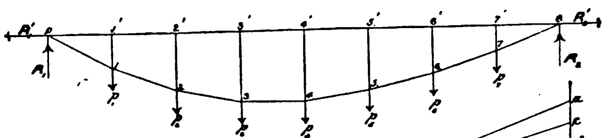
Exp., 25·7 „ 16·0 „

85. Show that the deflection at a point distant x from the centre of a beam loaded at the centre is $\left(l^3-(2x)^3\right)\dfrac{W.E.}{24\,M.h.}$

CHAPTER V.

THE ARCH.

49. *In Fig.* 41 *let the weights* $p_1, p_2, \ldots p_7$, *be sup-*



ported through the medium of the chain 0.1 . . . 8 at the points 0 and 8 by the reactions R_1R_1' *and* R_2R_2'; *to find the position of equilibrium by the graphic method.*

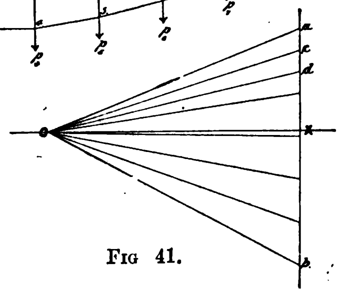


FIG 41.

To find the position of the line of equilibrium we have—

$$R_1.8 = p_1.7 + p_2.6 + \ldots + p_7.1,$$
$$\text{and } R_2.8 = p_1.1 + p_2.2 + \ldots + p_7.7,$$

and from these equations R_1 and R_2 may be obtained $p_1, p_2, \ldots p_7$, being known.

Draw the horizontal line XO and the vertical line aXb, and make $bX = R_1$ and $aX = R_2$; set off $p_1, p_2, \ldots p_7$, between b and a; take any point O on OX and join Oa, Oc, . . . etc.

Draw *8.7* parallel to aO, *7.6* parallel to cO, etc., and *0.1.2*, . . . *8* will be one of the lines of equilibrium.

It is manifest that there are an infinite number of curves of equilibrium each depending on the position of O.

When $p_1, p_2, \ldots p_7$ are all equal, the curve is a parabola, as can be proved by conic sections; but when *0.1*, *1.2* . . *7.8* are equal instead of *0.1′*, *1′.2′*, . . *7′.8′*, it is a catenary.

50. *The curve may also be found by the method of moments, and this is the more convenient way when we wish to make it pass through any fixed point.*

Suppose we wish it to pass through the point *3*, we have by taking moments about *3′* and putting S for the horizontal strain:—

$$R_1 3. - p_1.2 - p_2.1 = S.\ 3.3'.$$

The value of S being obtained from this equation, the value of *1.1′*, *2.2′*, etc., can be found by taking moments in the same way about *1′*,*2′*, etc.

51. In the case of an arch, the position of the curve of equilibrium may be obtained in the way given in either of the last two sections, but it will be above the horizontal line *0.8*, and the strains will be compressive instead of tensile.

52. With a brick or stone arch, the material of which is not capable of resisting a tensile strain, the line of resistance must fall at all points within the middle third of the thickness, as has been shown in section (21); but in one of timber or metal it may fall outside altogether.

The curve of equilibrium has an infinite number of positions; but in the former case, that which agrees nearest to the centre line of the arch should be taken, and in the latter the position is absolutely fixed owing to the effect that the strain has on the various members. This effect we will now proceed to consider.

53. *We shall prove that if the arch is tied or hinged at the abutments—* ΣMy *will be zero.*

If it is hinged at one abutment and tied at the other— ΣMy „ „ *and* ΣMx „ „

And if it is fixed at both abutments— ΣMy „ „ *and* ΣM „ „

Where ΣMy represents the sum of all the moments of the external forces each multiplied by the ordinate of the curve at its point of application, and so for ΣMx and ΣM, x and y being the co-ordinates of the curve referred to aa_n, as horizontal axis, and a as origin.

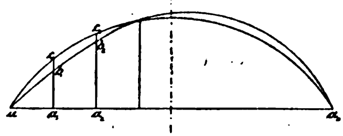


FIG. 42.

This may be easily explained by reference to Fig. 42, in which

$$\Sigma My = c_1b_1 \times c_1a_1 + c_2b_2 \times c_2a_2 + \; . \; . \; . \; . \; .$$
$$\Sigma Mx = c_1b_1 \times aa_1 + c_2b_2 \times aa_2 + \; . \; . \; . \; . \; .$$
$$\Sigma M \;\; = c_1b_1 \qquad + c_2b_2 \qquad + \; . \; . \; . \; . \; .$$

We have taken the verticals c_1b_1, c_2b_2, . . . instead of the normals to the curve, for it amounts to the same thing whether we take the thrust by the normal or the horizontal component by the vertical cb.

The position of the curve which will satisfy these equations may be found by trial with sufficient accuracy for all practical purposes.

54. *To prove these relations we may proceed as follows.*

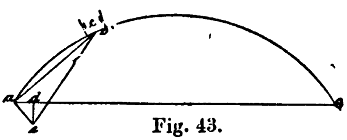


Fig. 43.

Take a short length c of the arch abg (Fig. 43), and let b be the centre point of it; call $ag = a$, and join ba.

Now, if ba is rigidly connected with c, when the arch is strained and the direction of c altered by the strain, ba will move through an angle $\Delta\theta$ to the position be; $\Delta\theta$ being the circular measure of the angle abe.

Now by drawing ae at right angles to ab, and ed at right angles to ag, it will be seen that the effect of the strain is to cause a horizontal displacement of ad, and a vertical displacement of ed.

We have by similar triangles, the angles bag, aed, being equal,

$$\frac{ad}{ae}=\frac{y}{ab}, \text{ or } ad=\frac{ae}{ab}.y=\Delta\theta.\ y,$$

and
$$\frac{ed}{ae}=\frac{x}{ab}, \text{ or } ed=\frac{ae}{ab}.x=\Delta\theta.\ x.$$

Now, since the arch is tied at a and g, the sum of all the horizontal displacements, caused by the alteration in direction of each length c, is zero,

Hence we have $\quad \Sigma y\Delta\theta \quad =0 \quad . \ . \ . \ (1)$

Next let α and β be the angles denoting the change in the direction of the arch at the abutments.

The arch at g, moving through the angle β, will cause a vertical displacement of $a.\beta$ at a, and since the total vertical displacement is zero, we have,

$$\Sigma x\Delta\theta + a.\beta = 0,$$

and $\quad \Sigma(a-x)\Delta\theta + a.\alpha = 0.$

If the arch is fixed at g, $\quad \beta = 0,$

so $\quad \Sigma x\Delta\theta = 0 \quad . \ . \ . \ (2)$

If it is fixed at a, $\alpha = 0$, so

$$\Sigma(a-x)\Delta\theta = 0.$$

By addition we get $\quad \Sigma a\Delta\theta = 0,$

or, since a is constant, $\quad \Sigma\ \Delta\theta = 0 \quad . \ . \ . \ (3)$

Hence

if the arch is hinged at a and g $\Sigma y\Delta\theta=0$,

if hinged at a, and fixed at g, $\Sigma y\Delta\theta=0$, and $\Sigma x\Delta\theta=0$,

and if fixed at a and g, $\Sigma y\Delta\theta=0$, and $\Sigma\ \Delta\theta=0$.

We have now to find $\Delta\theta$.

In Fig. 44 let NN be the neutral line of the arch ABCD, NN being equal to c, and let O be the centre of curvature before strain, and O′ after strain; call AN = NB = h, and the circular measures of the angles COA, C′O′A, θ and θ'.

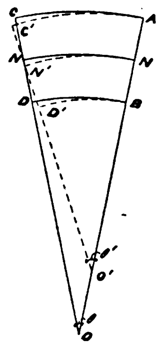


Fig. 44.

When strained the rib will assume the position ABC′D′, and NN that of NN′, NN′ being equal to NN and c.

Let f be the strain on the fibre AC, and A its sectional area.

Then the strain per unit on it will be $\frac{f}{A}$, and since a strain of $\frac{1}{E}$ (see end of section 29) would double the length of a bar under strain, if the law of elasticity held good to this extent, we have

$$\frac{fE}{A}=\frac{(c+\theta'h)-(c+\theta h)}{c+\theta h}=h\cdot\frac{\theta'-\theta}{c+\theta h},$$

or, $$f=h\cdot\frac{\theta'-\theta}{c+\theta h}\cdot\frac{A}{E}$$

$$=\frac{h\Delta\theta}{\theta(r+h)}\cdot\frac{A}{E}, \text{ calling } ON=r, \text{ and } \theta'-\theta=\Delta\theta.$$

The moment of this strain about NN will be

$$=\frac{h^2\Delta\theta}{\theta(r+h)}\cdot\frac{A}{E},$$

Hence $$M=\Sigma fh \qquad =\Sigma\frac{h^2\Delta\theta}{\theta(r+h)}\cdot\frac{A}{E}$$

$$=\frac{\Delta\theta}{\theta(r+h)}\cdot\frac{1}{E}\cdot\Sigma Ah^2,$$ for $\Delta\theta$, $\theta\,(r+h)$ and E are constant,

$$=\frac{\Delta\theta}{\theta(r+h)}\cdot\frac{I}{E},$$

$$=\frac{\Delta\theta}{\theta r}\cdot\frac{I}{E},$$ when, as is generally the case, h is small compared with r,

$$= \frac{\Delta\theta}{c}\cdot\frac{I}{E}\text{ since } c = \theta r,$$

or $$\Delta\theta = \frac{M.c.}{I}\cdot E. \qquad (4)$$

Substituting for $\Delta\theta$ in (1) (2) and (3) we have, since $\frac{c}{I}E$ is constant—for if the cross section is not uniform throughout, c may be so taken that $\frac{c}{I}$ is constant—

$$\Sigma y\,M = 0 \;\ldots\ldots\ldots\ldots \quad (5)$$
$$\Sigma x\,M = 0 \;\ldots\ldots\ldots\ldots \quad (6)$$
$$\Sigma M = 0 \;\ldots\ldots\ldots\ldots \quad (7)$$

If f' be the strain per unit of area on the most distant fibre from NN, or h', from it, then

$$f' = \frac{h'\Delta\theta}{\theta(r+h')}\cdot\frac{1}{E} = \frac{h'\Delta\theta}{c}\cdot\frac{1}{E},$$

when h' is small compared with r.

Combining this with (4) we have

$$M = \frac{f'}{h'}I. \;\ldots\ldots\ldots \quad (8)$$

55. *To find the horizontal thrust caused by a rise of temperature and the consequent moments on the arched rib.*

If the half arch ab (Fig. 45) is free, a rise of temperature, by extending it equally all round, will cause it to assume the position $a'b'$, and the horizontal displacement at a will be equal to $\left(\frac{l}{2}e.t\right)$, l being the whole span, t the rise of temperature in degrees, and e the extension per unit of length for each degree.

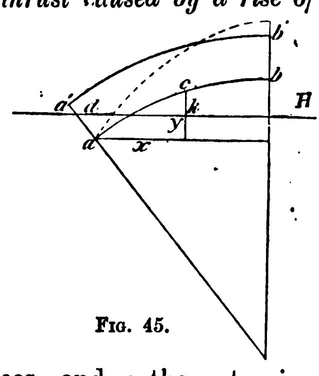


FIG. 45.

Let H be a horizontal force acting on the arch and producing a horizontal displacement of $\frac{l}{2}e.t$, that is to say, let H be the horizontal thrust caused by a rise of temperature in the arch when kept from spreading. It might perhaps at first sight be thought that the force H, as in Chap. I., sect. 3, would be 5 tons per square inch, *i.e.*, 5 tons multiplied by the sectional area of the rib. Further consideration, however, will make it clear that the arch will act as a long column, as in Chap. VI., and will oppose the pressure by its resistance to bending, and not by its direct resistance to crushing as if it was a short straight column. If the arch be hinged at a, H must act through a; but if it be fixed, H will act through some point d, ad above a.

The effect that the force H has at any point c, whose co-ordinates referred to a as origin are x and y, is to produce a bending moment M on the arch equal to H.kc, or if the arch be hinged, of H.y.

Now since the bending moments produce a horizontal displacement of

$$\Sigma y.\frac{Mc}{I}.E \quad \ldots\ldots\ldots \text{(1) and (4)}$$

we have
$$\frac{l}{2}e.t = \Sigma_b^a y.\frac{Mc}{I}.E,$$

which, since $M = H.kc$,
$$= \frac{Hc}{I}E\,\Sigma_b^a y.kc,$$

or
$$H = \frac{l}{2}e.t\frac{I}{E.c}\frac{1}{\Sigma_b^a y.kc}, \quad \ldots (9)$$

or, if the arch be hinged,
$$= \frac{l}{2}e.t\frac{I}{E.c}\frac{1}{\Sigma_b^a y^2}, \quad \ldots (10)$$

The vertical displacement v is $\Sigma_b^a x\frac{Mc}{I}E,$

or $$v = \frac{Hc}{I} E \Sigma_b^a x.kc.$$

Combining this with (9) $$v = \frac{l}{2} e.t \frac{\Sigma_b^a x.kc}{\Sigma_b^a y.kc} \quad \ldots\ldots (11)$$

or with (10) $$= \frac{l}{2} e.t \frac{\Sigma_b^a xy}{\Sigma_b^a y^2} \quad \ldots\ldots (12)$$

The maximum value of $e.t$, at any rate in this climate, may be taken as ·0005, so that (9) becomes

$$v = \frac{l}{2} \times \cdot 0005 \frac{\Sigma_b^a xy}{\Sigma_b^a y^2}.$$

And for a span of 100 ft. and rise of 20 ft.,

$$v = \frac{1200}{2} \times \cdot 0005 \times \frac{1034}{533},$$
$$= \cdot 58.$$

Examples.

1. A brick arch 42 ft. 6 in. span, 10 ft. rise, is 3 ft. thick at the abutments, and 2 ft. 3 ins. at the centre. The inclination of the roadway over it is 1 in 20, and there is a foot of road material over the crown; find the vertical and horizontal components of the reaction at each abutment per foot in width. *Answer*, Vertical, 7 tons and 8 tons.
Horizontal, 6 tons.

2. Find the reactions in the last case if the inclination on the higher side is 1 in 20 and on the lower 1 in 7.
Answer, Vertical, 6 tons and 8 tons.
Horizontal, $5\frac{1}{2}$ tons.

3. In Example 1 find the maximum strain per square inch on the brickwork. *Answer*, 83 pounds.

4. Will these arches stand? and, if not, why not?

5. One ton weights are suspended by a chain at horizontal distances of 10 ft. between each and between the last and the abutments, which are 100 ft. apart. If the deflection is 10 ft. at 40 ft. from one abutment, how much will it be at 10 ft., 30 ft., and 50 ft.?

Answer, 3 ft. 9 ins., 8 ft. 9 ins., and 10 ft. 5 ins.

6. Find the strain at the centre and at the abutments in Example 5, and also the horizontal and vertical reactions given by them. *Answer*, 12·012, 12·816, 12 and $4\frac{1}{2}$ tons.

7. A segmental arch, 50 ft. span and 10 ft. rise, is loaded at the centre; how far above the crown will the line of resistance be at the centre (*a*) when the arch is hinged, and (*b*) when it is fixed at the abutments.

Answer, (*a*) 3 ft., (*b*) 2 ft. at centre, 1 ft. 8 ins. at abutments.

8. Give the result in the last case when the arch is hinged if the load is equally distributed on the horizontal line.

Answer, 3 ins.

9. How much would a rise of temperature of 45° C. move an arch, of 100 ft. span and 10 ft. rise, upwards at the centre?

Answer, 1·17 ins.

10. An arch of 100 ft. span and 20 ft. rise has weights placed on it at horizontal distances of 10 ft. Give the values of the different weights with reference to the central weight, in order that the line of resistance may agree with the curve. Give also the vertical and horizontal thrusts of the abutments.

Answer, W, 1·1 W, 1·25 W, 1·7 W.

Vertical thrust 5·55 W, horizontal 7 W.

11. A footbridge is 6 ft. 10 ins. wide and 70 ft. span, and is supported by two wrought iron ribs each 7 ft. rise and 1 ft. 9 ins. deep. The dead load being 12 tons, give the horizontal thrust, the vertical reaction of each abutment, and the maximum strain on the flanges in each of the following cases, neglecting the strains which arise from change of form and temperature:—

	Horizontal Reaction.	Vertical Reaction.	" "	Maximum Strain.
If $\frac{3}{8}$ of the span is covered by the rolling load	12	$3\frac{3}{4}$	$6\frac{3}{4}$	15
If $\frac{5}{8}$ " " "	17	$5\frac{1}{4}$	$8\frac{1}{4}$	16
If the whole " " "	22	9	9	12

12. In the last example how much will the crown rise and fall with the variations of temperature?

Answer, ·82 in.

13. Find the additional strain that this will throw on to the arch at the crown, supposing the ribs to have been fixed at the maximum temperature.

Answer, 5·2 on one flange and 4·0 on the other, *i.e.*, about 1 ton per square inch.

14. Assuming the mean compressive strain on the metal of the rib to be 5 tons per square inch, what additional strain will be thrown on it owing to its change of form?

Answer. The strain has just the same effect as the temperature, *i.e.*, one ton per square inch, for 5 tons produce an extension of ·0005.

15. An iron ring 5 in. internal and 7 in. external diameter and 1 in. square in section, connects two chains; find the tensile strain that may be put on these if the metal may not be strained to over 5 tons per square inch.

Answer, 1 ton.

16. A segmental arch of brickwork 25 ft. span 7 ft. rise and 1 ft. $10\frac{1}{2}$ ins. thick, is covered with ballast, the top of which is horizontal and 3 ft. above the soffit at the centre; if the width of the arch is 13 ft. 4 ins. and the rolling load 2 tons per lineal foot, find the horizontal thrust, the maximum distance between the line of thrust and the curve of the arch, and the maximum pressure on the brickwork, (*a*) when the arch is all loaded and (*b*) when the load is half on, both by the graphic method and by the method of moments in each case.

Answer.	Thrust per foot in width	(*a*) 73 cwt.	(*b*) 58 cwt.
	Divergence	(*a*) $1\frac{1}{8}$ ins.	(*b*) $7\frac{1}{2}$ ins.
	Pressure per square inch	(*a*) 56 lbs.	(*b*) 58 lbs.

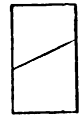
CHAPTER VI.

COMPRESSIVE STRAINS.

56. A column or strut under pressure may fail in three ways: firstly, by the metal being absolutely crushed; secondly, by the column bending and breaking near the centre of its length; and thirdly, by the plates composing it wrinkling, owing to their breadth being out of proportion to their thickness.

Failure by Crushing.

57. If the length is not many times more than the moment of resistance of the cross section of the column the failure will be simply by crushing, and the strength will depend only on the actual sectional area, with the exception that the resistance of thin metal per square inch is greater than that of thick.



With cast iron the failure is due to the upper part sliding, as it were, off the lower at an angle as in the wood-cut. The absolute resistance is variable, but may be taken at 42 tons per square inch, or 6 times the tensile resistance, and the safe resistance as 7 tons per square inch.

With wrought iron the metal will bulge or thicken out at the point of fracture, in such a way that it is difficult to state at what point actual failure takes place. For practical purposes, however, all materials may be considered to fail when their limit of elasticity

is exceeded. Now with wrought iron a pressure of each ton per square inch will produce a depression of ·0001 times the length until the strain is 10 or 12 tons, when the depression becomes more and more for each ton, and a permanent set is produced. In these pages 10 tons per square inch is taken as the limit of elasticity and 5 tons as the safe resistance, the actual weight under which complete failure ensues being from 16 to 20 tons.

With timber failure sometimes takes place by the fibres crushing into each other and sometimes by their splitting apart; in the latter case, although it may be due to absolute crushing, the length has some influence on the resistance, for each fibre may be considered to be a column failing by cross breaking, assisted however by its adhesion to the adjacent fibres.

The absolute resistance varies from 2 to 3 tons per square inch, and the limit of elasticity may be taken as about half this, and the safe resistance as 10 cwt. It is necessary, however, that the strains should act in the direction of the length, for if applied crossways the resistance is very much less; indeed the author has seen a weight of a ton, applied as in Fig. 46, shear off the fibres, although the length was 2 ft. and the width under pressure 6 ins., which indicates a resistance of only 15 pounds per square inch.

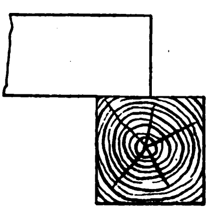


FIG. 46.

With stone and brick the failure is due to the weight first cracking the material, and then crushing it into powder. The resistances and safe loads for various substances are given in section 4, Chapter I.

FAILURE BY BENDING OR CROSS BREAKING.

58. *To find the weight which will cause a column to bend or break across.*

Let the weight W rest on a column half of which BB_n is shown in Fig. 47, and assume that it has deflected it to the extent shown, and let $D_1D_2 \ldots D_n$ be the deflections at $C_1B_1, C_2B_2, \ldots C_nB_n$.

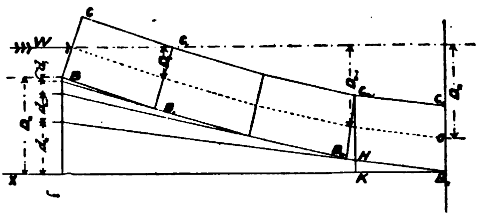


FIG. 47.

From C_{n-1} let fall the perpendicular $C_{n-1}K$ on B_nX cutting $B_{n-1}B_n$ in H.

At the section C_1B_1, we have $W.D_1 = s_1.M$,

s_1 being the strain induced, and M the Moment of Resistance of the section.

So at C_2B_2, we have $W.D_2 = s_2.M$,

. . . .

. . . .

and at C_nB_n, " $W.D_n = s_n.M$.

Now that portion which sustains a tensile strain of s_n will be lengthened to $a + s_n E.a$, a being the length of the short part under consideration.

We will consider the portion $C_{n-1}B_{n-1}.C_nB_n$ in which the strain is s_n, the bottom $B_{n-1}B_n$ being consequently lengthened from a to $a + s_n E.a$, and the top shortened to $a - s_n E.a$.

The strain, if we consider B_nC_n fixed, will cause $C_{n-1}B_{n-1}C_nB_n$ to revolve about the point O until $C_{n-1}C_n$ becomes $a - s_nE.a$ and $B_{n-1}B_n$ becomes $a + s_nE.a$, and then $B_{n-1}B_n$ will make an angle $B_{n-1}B_nX$ with B_nX, and this we will call θ_n.

Then since the angle $B_{n-1}HC_{n-1} = KHB_n$, and the angles $C_{n-1}B_{n-1}H$, HKB_n are right angles the angle $B_{n-1}C_{n-1}H = HB_nK = \theta_n$.

And so, calling the depth $2h$, we have

$$\tan\theta_n = \tan\ B_{n-1}C_{n-1}H = \frac{2\,s_n\,Ea}{2h} = \frac{s_n\,Ea}{h},$$

and also $\quad$ " $\quad = \dfrac{d_n}{na},$

$$\therefore \quad d_n = \frac{s_n\,Ea^2\,n}{h},$$

so $\quad d_{n-1} = \dfrac{s_{n-1}Ea^2\,(n-1)}{h},$

$$\vdots \qquad\qquad \vdots$$

$$d_1 = \frac{s_1\,Ea^2\,1}{h}.$$

Hence $\quad D_n = d_n + d_{n-1} + \cdots + d_1$

$$= \frac{Ea^2}{h}\{s_n\,n + s_{n-1}\,(n-1) + \ldots + s_1.1\}.$$

Now $\dfrac{D_n - D_{n-1}}{a} = \tan\ \theta_n = \dfrac{s_n\,Ea}{h},$

$$\therefore \quad D_{n-1} = D_n - s_n\frac{Ea^2}{h},$$

or $\quad s_{n-1}\dfrac{M}{W} = s_n\dfrac{M}{W} - s_n\cdot\dfrac{Ea^2}{h},$

$$\therefore \quad s_{n-1} = s_n\left\{1 - \frac{Ea^2}{h}\cdot\frac{W}{M}\right\}$$

In the same way

$$s_{n-2} = s_{n-1}\left\{1 - \frac{Ea^2}{h}\cdot\frac{W}{M}\right\},$$

$$= s_n\left\{1 - \frac{Ea^2}{h}\cdot\frac{W}{M}\right\}^2.$$

Hence $\quad D_n = \dfrac{Ea^2}{h}s_n\left\{n + (n-1)\left(1 - \dfrac{Ea^2}{h}\cdot\dfrac{W}{M}\right)\right.$

$$\left. + (n-2)\left(1 - \frac{Ea^2}{h}\cdot\frac{W}{M}\right)^2 + \cdots + 1.\left(1 - \frac{Ea^2}{h}\cdot\frac{W}{M}\right)^{n-1}\right\}.$$

Now in the expansions of $\left(1-\frac{Ea^2}{h}\cdot\frac{W}{M}\right)$ if n is very large a will be very small, and so everything above the second power of a may be neglected, and we have

$$
\begin{aligned}
D_n &= \frac{Ea^2}{h}s_n\Big\{[n+(n-1)+(n-2)+\ldots+1] \\
&\quad -\frac{Ea^2}{h}\cdot\frac{W}{M}[(n-1).1+(n-2).2+\ldots+1.(n-1)]\Big\}, \\
&= \frac{Ea^2}{h}s_n\Big\{[n+(n-1)+(n-2)+\ldots+1] \\
&\qquad -\frac{Ea^2}{h}\cdot\frac{W}{M}[(1+2+\ldots+\overline{n-1})\,n \\
&\qquad -(1^2+2^2+\ldots+\overline{n-1}^2)]\Big\}, \\
&= \frac{Ea^2}{h}s_n\Big\{\frac{n(n+1)}{2} \\
&\qquad -\frac{Ea^2}{h}\frac{W}{M}\left[\frac{n(n-1)n}{2}-\frac{(n-1)n(2n-1)}{6}\right]\Big\}
\end{aligned}
$$

Now since n is large $n+1$ and $n-1$ may be put $=n$, and we get

$$
\begin{aligned}
D_n &= \frac{Ea^2}{h}s_n\left\{\frac{n^2}{2}-\frac{Ea^2}{h}\cdot\frac{W}{M}\left[\frac{n^3}{2}-\frac{n^3}{3}\right]\right\} \\
&= \frac{E(na)^2}{h}s_n\left\{\frac{1}{2}-\frac{E(na)^2}{h.n}\frac{W}{M}\frac{1}{6}\right\}
\end{aligned}
$$

For na, $\frac{l}{2}$ may be put, l being the length of the column and $\frac{E(na)^2}{h.n}\cdot\frac{W}{M}\frac{1}{6}$ becomes $\frac{E\left(\frac{l}{2}\right)^2 W}{h.n.M.6}$, and may be neglected since n is very large.

Thus it appears that we might have neglected the second powers of a in the first instance, but this was not then quite clear.

We have therefore

$$D_n \quad = \frac{E\left(\frac{l}{2}\right)^2}{h} s_n \frac{1}{2} \quad = \frac{El^2}{8h} s_n.$$

Now $\quad W \quad = s_n \frac{M}{D_n} \quad = s_n \frac{M.8h}{El^2 s_n} = \frac{8Mh}{El^2} = \frac{8I}{El^2},$

$\therefore$ the strain per square inch

$$= \frac{W}{area} \quad = \frac{\frac{8I}{area}}{El^2} \quad = \frac{8r^2}{El^2},$$

$$= \frac{8}{E\left(\frac{l}{r}\right)^2},$$

that is to say, the resistance to bending depends only on E, and has no reference to the direct resistance to crushing.

59. To find the resistance of any column it is necessary first to find the radius of gyration, or r,

we have $r^2 \quad = \frac{I}{sectional\ area}.$

For a rectangular cross section

$$I = \Sigma b.ax^2,$$

$$= 2\frac{d}{2}\Sigma\frac{x}{\frac{d}{2}}.bax,$$

$$= d\frac{bd}{4}.\frac{d}{3} = \frac{bd^3}{12},$$

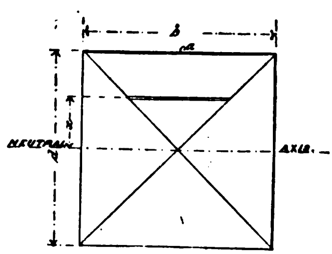


FIG. 48.

$$\text{Hence } r^2 = \frac{I}{\textit{sectional area}} = \frac{\frac{bd^3}{12}}{bd} = \frac{d^2}{12},$$

and $\quad r = d\sqrt{\cdot 0833},$

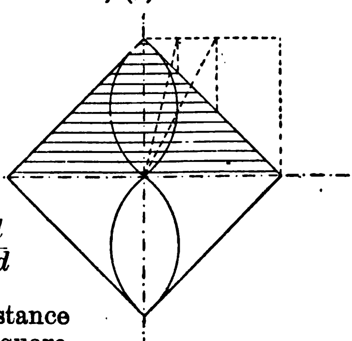
$\quad = d \cdot 288,$

$$\therefore \quad \frac{l}{r} = \frac{l}{d \cdot 288} = 3 \cdot 5 \frac{l}{d}$$

For a square placed diagonally

$$\begin{aligned} \mathrm{I} = 2 \times \cdot 10 \frac{d}{2} \Big\{ & \cdot 10 \times 1 \cdot 35^2 + \cdot 30 \times 1 \cdot 25^2 + \cdot 50 \times 1 \cdot 15^2 \\ & + \cdot 70 \times 1 \cdot 05^2 + \cdot 90 \times \cdot 95^2 \\ & + 1 \cdot 10 \times \cdot 85^2 + 1 \cdot 30 \times \cdot 75^2 \\ & + 1 \cdot 50 \times \cdot 65^2 + 1 \cdot 70 \times \cdot 55^2 \\ & + 1 \cdot 90 \times \cdot 45^2 + 2 \cdot 10 \times \cdot 35^2 \\ & + 2 \cdot 30 \times \cdot 25^2 + 2 \cdot 50 \times \cdot 15^2 \\ & + 2 \cdot 70 \times \cdot 05^2 \Big\} \left(\frac{d}{2}\right)^3 \end{aligned}$$

$$= \cdot 08 d^4.$$



$$\text{Hence } r^2 = \frac{I}{\textit{area}} = \frac{\cdot 08 d^4}{d^2},$$

and $\quad r = \cdot 283 d,$

$$\therefore \quad \frac{l}{r} = \frac{l}{\cdot 283 d} = 3 \cdot 53 \frac{l}{d}$$

Hence the diagonal resistance is nearly the same as the square.

FIG. 9.

For a solid circular section

$$\begin{aligned}I &= 2 \times \cdot 10\frac{d}{2}\Big\{\cdot 60 \times \cdot 95^2 + 1\cdot 05 \times \cdot 85^2 + 1\cdot 33 \times \cdot 75^2 \\ &\quad + 1\cdot 52 \times \cdot 65^2 + 1\cdot 66 \times \cdot 55^2 \\ &\quad + 1\cdot 80 \times \cdot 45^2 + 1\cdot 88 \times \cdot 35^2 \\ &\quad + 1\cdot 95 \times \cdot 25^2 + 2\cdot 00 \times \cdot 15^2 \\ &\quad + 2\cdot 00 \times \cdot 05^2\Big\}\left(\frac{d}{2}\right)^3, \\ &= \frac{\cdot 10d^4}{8} 3\cdot 96 = \cdot 05d^4,\end{aligned}$$

and $r^2 = \dfrac{I}{area} \quad = \dfrac{\cdot 05d^4}{\pi\left(\frac{d}{2}\right)^2},$

$$= \frac{\cdot 20}{3\cdot 1416}d^2,$$

or $r = \cdot 25d,$

$\therefore \quad \dfrac{l}{r} = \dfrac{l}{\cdot 25d} \quad = 4\dfrac{l}{d}$

For a thin hollow cylinder

$$I = 4 \times \cdot 1122d't. \Big\{\cdot 50^2 + \cdot 47^2 + \cdot 43^2 + \cdot 36^2 + \cdot 28^2 + \cdot 17^2 + \cdot 06^2\Big\}d'^2,$$

$$= \cdot 4488d'^3t \times \cdot 8963 = \cdot 40d'^3t,$$

and

$r^2 = \dfrac{I}{area} \quad = \dfrac{\cdot 40d'^3t}{3\cdot 1416d't},$

$$= \frac{\cdot 40}{3\cdot 1416}d'^2.$$

Hence

$r = d'\sqrt{\cdot 1273} \quad = \cdot 35d'.$

$\therefore \dfrac{l}{r} = \dfrac{l}{\cdot 35d'} \quad = 3\dfrac{l}{d'}.$

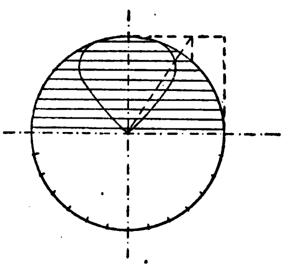


FIG. 50.

The diameter must be taken to the centre of the metal, hence it is evident that the thicker the metal the less will r be, and so the resistance per square inch will be less.

$$\text{Putting } t=\frac{d}{10}, \text{ we have } r=d'\cdot35=\cdot9d\cdot35=\cdot315d,$$

$$\text{or } \quad ,, \quad =\frac{d}{5}, \quad ,, \quad r \quad =\cdot8d\cdot35=\cdot28d.$$

60. It will be seen that no weight less than W will have any tendency to bend the column; and if it is deflected by a force applied at right angles to it, when the force is removed, the resistance being greater than the weight, the column will recover its position so long as the deflection is not carried too far. If however the deflection D_n is made so great that s_n becomes more than 10 or 12 tons, in the case of wrought iron, the value of E will become greater, and consequently the weight carried will be less. If, again, the load carried is greater than 10 or 12 tons per square inch, the value of E will also become greater, so that when the ratio of length to radius of gyration becomes so small that the column is on the point of failing by direct crushing, the resistance to bending will be less.

In practice, however, by no accident should the strain ever exceed 10 tons, and the usual working load is 5 tons per square inch.

61. A factor of safety of about $\frac{1}{4}$th the absolute resistance of the metal is commonly taken, but this is really only $\frac{1}{2}$ of the elastic resistance.

Now in a long column, until the breaking weight is actually reached, except in the cases just referred to, there is, theoretically, no tendency to bend, and certainly the elastic limit is not exceeded, hence so large a factor of safety is not required. Owing however to differences in the different parts of the same metal in the value of E, and also to the possible divergence of

the line of pressure from the neutral line, some margin must be allowed, and it is perhaps as well to take a factor of $\frac{5}{3}$, or to put it that the weight per square inch that may safely be put on a column is $\frac{3}{E\left(\frac{l}{r}\right)^2}$, and that this in the case of cast iron should not be more than 7 tons, of steel $6\frac{1}{2}$ tons, and of wrought iron 5 tons, for short columns will fail by crushing. A deduction for rivet holes must be made in calculating the sectional area.

With cast iron we have $E = \cdot 00018$, steel $E = \cdot 00008$, and wrought iron $E = \cdot 0001$. Hence, the maximum strain that should be put on the metal—

		Cast Iron.	Steel.	Wrought Iron.
when $\frac{l}{r}$ =	100, is	1·66 tons	3·75 tons	3·00 tons
,,	200 ,,	0·41 ,,	0·94 ,,	0·75 ,,
,,	300 ,,	0·18 ,,	0·42 ,,	0·33 ,,
,,	400 ,,	0·10 ,,	0·23 ,,	0·19 ,,

If a column is fixed at the ends, the strength is three times that of the unfixed column, and if fixed at one end and free at the other, the strength is twice that of the unfixed column, the strains acting as in a continuous girder (Fig. 51). In the first case the length between the points of inflection is a little more than half the length, and in the second a little more than two-thirds, for

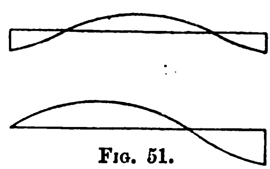


Fig. 51.

$$1 : 2 : 3 \quad :: \frac{l}{1} : \frac{l}{\cdot 50} : \frac{l}{\cdot 33}$$

$$:: \frac{l}{1^2} : \frac{l}{(\cdot 7)^2} : \frac{l}{(\cdot 58)^2}$$

With timber, the value of E may be taken as ·001, and a factor of safety of 6 should be allowed. Thus the safe load in tons per square inch is

$$\frac{4}{\cdot 003\left(\frac{l}{r}\right)^2} \text{ or } \frac{1333}{\left(\frac{l}{r}\right)^2}$$

which must not be more than 10 cwt.

THE WRINKLING STRAIN.

62. When the width of a plate is considerable compared with its thickness, failure may take place by the metal wrinkling or corrugating between the supports. By means of a number of experiments on square tubes in which the plates were from $\frac{1}{2}$ to $\frac{1}{4}$ inch thick, and 8 inches and 4 inches wide, and were supported at each side and the weight applied directly, and again by experiments on tubular girders, in which they were from $\frac{1}{32}$ to $\frac{3}{4}$ inch thick, and from 2 to 24 inches wide, the following empirical formula has been established—

$$s = 70\sqrt{\frac{t}{b}},$$

in which s is the resistance in tons per square inch of the metal to wrinkling, and t and b the thickness and breadth of the plate when supported at each edge.

It might perhaps be equally well expressed,

$$s = \sqrt{5{,}000\frac{t}{b}},$$

or taking 4·1 as the factor of safety,

$$s = \sqrt{300\frac{t}{b}}.$$

When a plate is supported at one edge only, its resistance must be considered to be not more than $\frac{1}{3}$ of that supported at both edges, and thus we have

$$s = \sqrt{100\frac{t}{b}}.$$

EXAMPLES.

1. Give the weights in tons per square inch which would break the following wrought iron columns, when fixed at the ends.

Length. ft.	ins.	Thickness. ins.	*Answer.* tons.	*Experiment* (Box, 141). tons.
2	6	$\frac{3}{4}$	12·2	12·4 for ·76 thickness.
2	6	$\frac{1}{2}$	5·4	7·5
5	0	1	5·4	7·7, 8·0, and 7·9
7	6	$1\frac{1}{2}$	5·4	8·9
5	0	$\frac{3}{4}$	3·1	5·8 for ·77 thickness.
10	0	$1\frac{1}{2}$	3·1	4·5
7	6	1	2·4	4·4, 4·4 and 4·1
5	0	$\frac{1}{2}$	1·4	2·5
10	0	1	1·4	1·9

2. Also of the following, when not fixed at the ends.

Length. ins.	Thickness. in.	*Answer.* tons.	*Experiment* (Author). tons.
$7\frac{1}{2}$	$\frac{1}{4}$	7·3	7·5
$10\frac{1}{2}$	"	3·7	6·4
12	"	2·8	4·0
15	"	1·8	3·5
18	"	1·3	2·1

3. Give the breaking weights of the following 1 inch diameter wrought iron cylindrical columns when fixed at both ends, one end, and neither end, in tons per square inch.

		Fixed at both ends.		Fixed at one end.	
Length. ft.	ins.	*Answer.* tons.	*Exp.* (Box, 124). tons.	*Answer.* tons.	*Experiment.* tons.
1	3	16·0	15·3 and 15·3	16·0	13·8 and 15·8
2	6	16·0	12·8 and 14·0	11·1	11·8, 12·5 and 10·8
5	0	4·2	7·1	2·8	4·4
7	6	1·8	2·9	1·2	1·8

Rounded at both ends.

Length. ft.	ins.	*Answer.* tons.	*Experiment.* tons.
1	3	16·0	13·4 and 13·2
2	6	5·5	9·0, 8·4 and 8·2.
5	0	1·4	2·2 and 2·1
7	6	0·6	1·0 and 1·0

4. Give the breaking weights per square inch of the following wrought iron tubes fixed at both ends.

Length. ft.	in.	External Diam. in ins.	Thickness. in.	*Answer.* tons.	*Experiment* (Box. 139). tons.
9	9	4·05	·15	16·0	11·7 and 12·3
9	6	3·00	·15	16·0	12·4
10	0	2·50	·10	10·6	13·3
9	11	2·35	·25	8·3	9·6 and 9·9
10	6	2·00	·10	6·0	10·3
10	5	1·50	·10	3·3	6·5

5. Give the breaking weight in tons per square inch of the following solid cylindrical pillars of cast iron, both with the ends flat and with them rounded.

	Ends fixed.				Ends rounded.			
$\frac{l}{d}$	*Answer.*	*Exp.* (Box, 108). diam. in ins.			*Answer.*	*Experiment.* diam. in ins.		
		$\frac{1}{2}$ or $\frac{3}{4}$.	1.	$1\frac{1}{2}$ or 2.		$\frac{1}{2}$ or $\frac{3}{4}$.	1.	$1\frac{1}{2}$ or 2.
10	42·0	30·3		—	27·7	21·7	—	—
15	37·0	25·6	22·9	—	12·3	12·0	11·3	—
20	21·0	—	—	—	7·0	—	—	8·0
30	9·2	15·4	11·4	—	3·1	4·3	3·5	3·6
40	5·2	—	—	6·7	1·7	—	—	2·7
60	2·3	3·7	3·5	—	0·8	1·2	1·1	—
120	0·6	1·1	—	—	0·2	0·3	—	—

6. Give the breaking weights in tons per square inch of the following hollow cast iron cylinders, as before.

Length. ft.	in.	Ext. Diam. Inches.	Thickness. Inches.	Fixed at ends. *Answer.*	*Exp.* (Box, 108).	Ends rounded. *Answer.*	*Exp.*
0	9	1·11	·11	42·0	43·1	—	—
1	3	„	„	„	32·0	—	—
2	0	1·16	·20	23·7	23·0	—	—
2	6	1·25	·25	16·5	19·1	—	—
4	9	1·75	„	—	„	3·4	4·6
7	6	„	„	4·1	6·0	1·4	1·9
„		2·00	·30	5·4	7·9	1·8	2·3
„		2·25	·35	6·7	8·9	—	—
„		„	·25	—	—	2·4	3·8
„		2·50	·30	—	—	2·9	4·3
„		2·75	„	—	—	3·7	5·4
„		3·00	·25	—	—	4·6	5·2
„		3·35	·35	—	—	5·5	6·6

7. Give the breaking weight in tons per square inch of the following rectangular pieces of wood rounded at both ends.

Ratio of Length to Thickness.	*Answer.*	*Experiment* (Author). Pitch Pine. $\frac{1}{4}$ in. & $\frac{3}{8}$ in.	$\frac{1}{2}$ in. & $\frac{3}{4}$ in.	1 in. & $1\frac{1}{2}$ in.	Red Deal. $\frac{1}{4}$ in. & $\frac{3}{8}$ in.	$\frac{1}{2}$ in. & $\frac{3}{4}$ in.	1 in. & $1\frac{1}{2}$ in.
8	3·00	—	2·9	—	—	2·5	—
16	2·50	—	1·6	2·0	—	—	1·8
18	2·00	—	—	2·0	—	—	1·3 / 1·4
24	1·10	—	1·5 / 1·3	1·7 / 1·5	—	1·3	1·3
32	0·64	0·55	0·7	—	0·50	0·7	—
$37\frac{1}{2}$	0·46	—	0·6	—	—	0·6	—
50	0·26	0·28	0·3	—	0·24	0·3	0·2

8. Find the radius of gyration of the strut shown in Fig. 52.

Answer, 3·7 ins. and 3 ins.

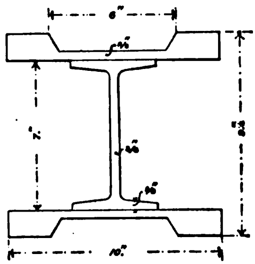


Fig. 52.

9. Give the breaking weight of the strut in example 8, if the ends are rounded, when $\frac{l}{r}$ is (*a*) 80, and (*b*) 100.

	(*a*)		(*b*)	
Answer,		12·5		8·0
Exp. (Min. Inst. C.E. 54–201)	17	11·8	14	9·8
	18	14·0*	16	11·9
			19	12·4*

10. Find the breaking weight of a wrought iron tube $8\frac{3}{8}$ in. external diameter and $\frac{3}{8}$ in. thick, having four 1-in. flanges projecting $1\frac{1}{2}$ ins. outside; 27 ft. long.

Answer, 6·8 tons.
Exp., 9·7 „

11. Give the breaking weight of the column shown in Fig. 53, if it is 25 ft. long, and the ends are rounded.

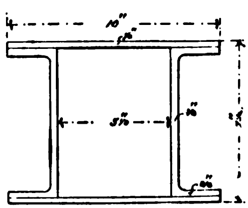


Fig. 53.

* Ends flat.

Answer,		8·0 tons.
Exp. 11	(Min. Inst. C.E. 54-202)	11·4 „
22	„ „ „	13·4 „
23	„ „ „	14·7 „

12. An octagonal column 12 in. diameter and 27 ft. long is formed of $\frac{1}{4}$ in. iron with four external joints each $1\frac{1}{2}$ in. deep. Find the breaking weight in tons per square inch, the ends being hinged. *Answer,* 13·8.
Exp. (Min. Inst. C.E. 54-203), 9·8.*

13. In what direction is a strut of + section most likely to fail?

14. What weight per square inch will break a cast iron strut of this section 3 ins. × 3 ins. × $\frac{1}{2}$ in., 7 ft. 6 ins. long and rounded at the ends. *Answer,* 2·25 tons.
Exp. (Box, 118) 3·00 „

15. What weight per square in. will break a 3 in. × $2\frac{1}{4}$ in. × ·35 in. cast H I. 7 ft. 6 ins. long and free at the ends? *Answer,* 3·25 tons.
Exp. (Box, 120) 5·00 „

16. Find the breaking weights of the following wrought iron struts, each of which is 5 ft. long and rounded at the ends, in tons per square inch.

Inches.			
$3 \times 2\frac{1}{2} \times \frac{3}{8}$ + I.	*Ans.*, 5·5	*Exp.* (Box, 143)	9·0 & 8·2
$3 \times 3 \times \frac{3}{8}$ T I.	„ 9·0	„ „	13·6 & 9·1 (sideways)
$3 \times 3 \times \frac{5}{16}$ L I.	„ 7·4	„ „	8·8 & 5·8
$3 \times 1\frac{3}{4} \times \frac{3}{8}$ [I.	„ 5·5	„ „	8·0 & 6·5

17. Give the results when the L I. in the last question is 4 ft. and when it is 3 ft. long. *Answer,* 11·7 and 16·0.
Exp., 11·1 and 12·9.

18. Give the maximum value that may be given to $\frac{l}{r}$ in order that taking $\frac{8}{3}$ as a factor of safety for cast iron, steel and wrought iron, and 6 for timber, a pressure of 7 tons, $6\frac{1}{2}$ tons, 5 tons, and 10 cwt. may be put on a column of cast

---

* There were 12 other experiments on similar columns with flat ends, and these confirm the result. Exp. 33 (Min. Inst. C.E. 54-212) also bears on the results in the last five examples.

iron, steel, wrought iron, and wood respectively, both when the ends are rounded and when they are fixed.

	Cast iron.	Steel.	Wrought iron.	Timber.
Answer, Ends Rounded,	49	76	78	52
" Fixed,	85	131	135	90

19. What weight may be safely placed on a timber strut 8 ft. 8 ins. long and 4 ins. square, if the ends are (a) fixed and (b) not fixed? *Answer*, (a) 7·7 tons; (b) 2·5 tons.

20. Give the weights if the strut is 8 ins. wide, 4 ins. thick, and the same length as before.

Answer, (a) 15·4 tons; (b) 5·1 tons.

21. To what length may a 6 in. × 6 in. strut be carried without reducing its strength to less than the maximum of 10 cwt. per square inch? *Answer*, 7 ft. 5 ins.

22. A weight of 11 tons is to be carried by a square prop 6 ft. 6 ins. long; give its proper size.

Answer, 5 ins. × 5 ins.

23. Each pier of a double line bridge consists of six 10 in. × 10 in. piles, the spans being 20 feet. Find the greatest length that should be given to the piles without bracing.

Answer, 17 ft. 8 ins.

24. A screw pile is 4 inches in diameter, and of wrought iron. At what distances should it be braced in order that it may sustain a load of (a) 5 tons, and (b) $2\frac{1}{2}$ tons per square inch, the ends being assumed to be fixed?

Answer, (a) 11 ft. 3 ins.; (b) 15 ft. 11 ins.

25. Find the resistance in tons per square inch which the following square tubes offer, the dimensions being internal.

Inches.		*Answer.*	*Exp.* (Box, 150).
$8 \times \frac{1}{4}$	thick.	12·3	11·5, 12·0, and 13·4
$8 \times \frac{1}{8}$	"	8·8	9·1
$8 \times \frac{1}{16}$	"	6·2	5·9, 5·9, 6·8, 7·2, and 7·1
$4 \times \frac{1}{8}$	"	12·3	9·6, 10·4, and 13·6
$4 \times \frac{1}{12}$	"	10·1	11·2, 10·5, and 13·2
$4 \times \frac{1}{16}$	"	8·8	8·8, 10·3, 10·4, 10·7, 9·9, 9·8, 11·6 and 8·6
$4 \times \frac{1}{32}$	"	6·2	4·9, 5·1, and 5·5

26. Find the pressure which would wrinkle the following plates when supported at each side.

Breadth. ins.	Thickness. ins.	*Answer.*	*Exp.* (Box, 229).
$22\frac{1}{2}$	$\frac{3}{4}$	12·8	$13\frac{1}{4}$ and $17\frac{1}{2}$
$22\frac{1}{2}$	$\frac{1}{2}$	10·5	16 and $15\frac{1}{2}$
15	$\frac{3}{4}$	15·6	16 and 15
15	$\frac{1}{2}$	12·8	18, 18, and 15
15	$\frac{1}{4}$	9·0	$14\frac{1}{2}$
15	$\frac{1}{8}$	6·4	$7\frac{1}{2}$
$3\frac{3}{4}$	$\frac{1}{8}$	12·8	$21\frac{1}{2}$
$3\frac{3}{4}$	$\frac{1}{16}$	9·0	15
$1\frac{7}{8}$	$\frac{1}{16}$	12·8	23
$1\frac{7}{8}$	$\frac{1}{32}$	9·0	$12\frac{1}{2}$

27. Give the resistance to wrinkling of the following square tubes in tons per square inch, the dimensions being internal.

Square Inches.	Inches.	*Answer.*	*Experiment* (Author). Square.		*Experiment.* Round.
1 × 1	·03 thick	12·1	14·3	1 in. diam.	24·5
1 × 1	·04 „	14·0	11·1	„ „	21·0
2 × 2	·03 „	8·5	6·7*	2 in. „	10·7
2 × 2	·04 „	9·9	7·0*	„ „	12·5

28. Give the resistance to wrinkling of the following ⊏.I's in tons per square inch:—

Inches.	*Answer.*	*Experiment* (Author). Load at Web.†
1 × 3 × ·08	6·6	10·6
1 × 3 × ·04	4·6	5·8
1 × 3 × ·03	4·0	5·0
$1 \times 2\frac{1}{2} \times$ ·03	4·4	5·5 and 2·0‡
1 × 2 × ·08	8·1	12·8 and 5·1‡
1 × 2 × ·04	5·7	7·5
1 × 2 × ·03	5·0	7·7
$1 \times 1\frac{1}{2} \times$ ·08	9·3	18·2
$1 \times 1\frac{1}{2} \times$ ·04	6·6	7·8
$1 \times 1\frac{1}{2} \times$ ·03	5·7	12·0
1 × 1 × ·12	14·0	13·5‡

* In these cases only two of the sides gave way. Hence if the load had been properly centred the resistance would probably have been greater.

† In these experiments the 1 inch web which was supported at each end carried more than its proper proportion of the load. However, in the case of flanged girders, this is what occurs in practice, the strain being transmitted to the flange at its centre.

‡ In these cases the load was applied at the centre of gravity.

29. Three warehouse floors are supported by cast iron columns, placed 20 feet apart, 15 feet long. If the columns are hollow and considered to be free at both ends, give their external diameter and thickness.

	Diameter.	Thickness.
Answer,	13 ins.	$\frac{3}{4}$ in.
If fixed at both ends and of wrought iron plate. . . .	12 "	$1\frac{1}{4}$ "

30. Give the greatest widths which may be given to the following plates in order that they may not fail by wrinkling, the resistance to crushing being taken as 16 tons per square inch.

	$\frac{1}{4}$ in.	$\frac{3}{8}$ in.	$\frac{1}{2}$ in.	$\frac{5}{8}$ in.	$\frac{3}{4}$ in.
Answer,	5	$7\frac{1}{2}$	10	$12\frac{1}{2}$	15

31. In the top flange of a girder what is the greatest pitch which should be given to the rivets, when the plates are (*a*) $\frac{1}{4}$ in. and (*b*) $\frac{1}{2}$ in. thick? *Answer*, (*a*) $9\frac{1}{2}$ ins., (*b*) 19 ins.

32. Find the radius of gyration of the following angle irons:—

	$1 \times 1 \times \frac{1}{8}$:	$1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{4}$:	$3 \times 3 \times \frac{1}{4}$:	$3 \times 3 \times \frac{1}{2}$:	$4 \times 3 \times \frac{1}{2}$.
Ans.,	·20 in.	·28 in.	·60 in.	·56 in.	·66 in.

33. Also of the following **T.** irons:—

	$1\frac{1}{2} \times 1\frac{1}{2} \times \frac{3}{16}$:	$2 \times 2 \times \frac{1}{4}$:	$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$:	$3 \times 3 \times \frac{1}{2}$:	$4 \times 4 \times \frac{1}{2}$.
Ans.,	·32 in.	·42 in.	·55 in.	·62 in.	·84 in.

34. Find the breaking weights of the following round or hinged ended **L.I.** and **T.I.** struts:—

			Answer.	*Exp.* (Christie).*
$3 \times 3 \times \frac{3}{8}$ in.	**L.I.**	8 ft. 4 ins. long.	2·7 tons.	6·0 tons.
$2 \times 2 \times \frac{5}{16}$ "	"	8 " 3 " "	1·3 "	1·7 " 3·0† "
$1 \times 1 \times \frac{1}{8}$ "	"	5 " 3 " "	0·8 "	1·3 " 2·5 "
$3 \times 3 \times \frac{1}{2}$ "	**T.I.**	8 " 4 " "	3·0 "	6·0 "
$1\frac{1}{2} \times 1\frac{1}{2} \times \frac{3}{16}$ "	"	8 " 3 " "	0·8 "	2·1 "
$1 \times 1 \times \frac{3}{16}$ "	"	5 " 3 " "	0·9 "	2·1 "
$3 \times 3 \times \frac{1}{2}$ "	"	6 " 10 " "	4·6 "	6·0 " 7·0 "
$1\frac{1}{2} \times 1\frac{1}{2} \times \frac{3}{16}$ "	"	5 " 3 " "	2·0 "	3·5 "

* *American Society of Civil Engineers*, vol. xiii. p. 85.
† When properly centred.

35. Also of the following fixed-end Angle irons :—

				Answer.	Exp. (Christie).*
$4 \times 4 \times \frac{3}{8}$ in.	L.I.	14 ft.	6 ins. long.	5·1 tons.	7·1 tons. / 6·0 „
$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$ „	„	14 „	6 „ „	2·0 „	3·5 „ / 2·8 „
$2 \times 2 \times \frac{5}{16}$ „	„	4 „	6 „ „	13·2 „	10·0 „

36. Also of the following flat-ended Tee irons :—

				Answer.	Exp. (Christie).
$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$ in.	T.I.	5 ft.	0 ins. long.	6·6 tons.	12·6 tons.
$2 \times 2 \times \frac{1}{4}$ „	„	6 „	6 „ „	2·3 „	5·7 „
$1\frac{1}{2} \times 1\frac{1}{2} \times \frac{3}{16}$ „	„	8 „	0 „ „	0·9 „	4·0 „ / 2·8 „
$1 \times 1 \times \frac{3}{16}$ „	„	6 „	6 „ „	0·6 „	1·2 „
$1 \times 1 \times \frac{3}{16}$ „	„	8 „	0 „ „	0·4 „	1·2 „ / 0·9 „

37. Also of the following flat-ended Angle irons :—

				Answer.	Exp. (Christie).
$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ in.	iron	10 ft.	6 ins. long.	2·3 tons.	7·2 tons.
$2 \times 2 \times \frac{5}{16}$ „	„	3 „	6 „ „	7·3 „	10·7 „ / 12·7 „
$1 \times 1 \times \frac{1}{8}$ „	„	5 „	0 „ „	0·9 „	3·1 „
$3 \times 3 \times \frac{5}{16}$ „	steel,	14 „	1 „ „	1·3 „	4·0† „
$3 \times 3 \times \frac{5}{16}$ „	„	7 „	0 „ „	5·1 „	11·3 „
$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ „	„	3 „	6 „ „	14·3 „	15·2 „

38. What weight in tons per sq. in. may safely be placed on 3 in. × 3 in. × $\frac{3}{8}$ in. L.I. struts, 6 ft. long and 8 ft. long?

Answer, 2·0 and 1·10.

39. Also on 3 ins. × 3 ins. × $\frac{1}{2}$ in. T.I. struts?

Answer, 2·2 and 1·25.

40. Give the weight in tons that can safely be put on $2\frac{1}{2}$ in. × $2\frac{1}{2}$ in. × $\frac{1}{2}$ in. L.I. struts 6 ft. long and 8 ft. long respectively, when the ends are fixed.

Answer, 9·0 and 5·0.

---

* *American Society of Civil Engineers*, vol. xiii. p. 85.
† „ „ „ „ „ p. 264.

41. Find the breaking weight of a $\frac{3}{4}$ in. × $\frac{3}{4}$ in. × $\frac{1}{8}$ in. L.I. strut, if the ends are rounded, when 12 ins., 18 ins. and 24 ins. long.

	12 ins.	18 ins.	24 ins.
Answer	1·85	0·82	0·46
Exp. (author), Flat ends	2·30	1·70	1·50
	2·35	1·60	1·00
Same pieces straightened hot	2·35	1·60	1·00
	2·15	1·30	1·00
Ends rounded as with one rivet in one member	1·65	0·95	0·60
	—	0·92	0·57

NOTES ON THE EXAMPLES.

CHAPTER IV.

4. The extreme fibres are strained somewhat beyond the limit of elasticity.

33. $M = 2{\cdot}49 \times 3{\cdot}5$ (Baker, 64).

37. We may put $s = 20$ for the flanges and 18 for the web.

40. We may take the whole of the flange—viz., 4 ins. × $\frac{1}{4}$ in.—as exposed to the tensile strain, and $\frac{1}{6}$th of this area at the extreme end of the web as exposed to compression, and consider the remainder of the web as not strained at all. Thus we have $s\,M = 6\frac{1}{2}$ (4 ins. × $\frac{1}{4}$ in.) $\cdot 90$ or $= 39\ (\frac{1}{6} \times \cdot 90)$.

41. Here we should take the whole of the web as exposed to tension, and we have $s\,M = 6\frac{1}{2}\ (1{\cdot}10 \times \cdot 25)\ (\cdot 55 + \cdot 25)$.

It seems in 40 that the whole of the flange does not quite give out the full unit strain, but in this case it appears, from the excess of the experiment over the result, that the flange does give out some resistance to the tensile strain.

44. The neutral axis is 10 ins. from the top and $M = 6{\cdot}9 \times 10{\cdot}8$.

46. The neutral axis is ·85 from the top of the web

so $\qquad M = \cdot 104\ [\cdot 567 + \cdot 288], = \cdot 09.$

47. $\qquad s\,M = 22\ (\cdot 21 \times \cdot 7)$

48. The neutral axis is 1·30 from the top of the web

so $\qquad M = \cdot 245\ [\cdot 555 + \cdot 867], = \cdot 35.$

49. If the whole of the flange is exposed to tension, a depth of 1·33 in. of the web will be exposed to compression. Hence approximately

$$s\,M = 22 \text{ (2 ins.} \times \tfrac{1}{4} \text{ in.)}\ [\cdot 125 + 1{\cdot}085] = 22 \times \cdot 6.$$

50. We may take the bottom flange together with a certain

depth x of the webs as exposed to the full strain of 7 tons per square inch. Thus we have

$$W=\frac{4}{l}sM=\frac{7}{18}\left\{\frac{2\cdot21\times3}{8}3\cdot82+2x\frac{3}{8}\left(3\cdot60-\frac{7x}{12}\right)\right\},$$

and putting 2·3 for W, we get $x=1\cdot3$.

This gives a depth of about 2 ins. for which we have allowed no resistance, and from this we must infer that the portion of x nearest to the neutral axis does not give out the full strain of 7 tons; but we do not know the exact tensile resistance of the metal, so cannot establish a theory.

51. Here we have as before

$$W=\frac{7}{18}\left\{\frac{25}{8}\frac{3}{8}\frac{93}{32}+2\frac{3}{8}x\left(\frac{43}{16}-\frac{7}{12}x\right)\right\},$$

whence $x=1\cdot65$.

53. We have as in sec. 29:—$\tan\theta=\frac{2\delta a}{h}$ and also $=\frac{d_n}{na}$.

Hence $d_n=\frac{2\delta a}{h}na$, $\therefore D=d_1+d_2+\ldots+d_n=\frac{2a\delta a}{h}(1+2+\ldots+n)$

$$=2a\frac{\delta a}{h}\frac{n^2}{2}\qquad=\frac{a^2n^2}{h}\frac{\delta a}{a}$$

Now $an=\frac{l}{2}$ and $\frac{\delta a}{a}=sE=\frac{Wl}{4M}E$. Hence $D=\frac{l^2}{4h}\cdot\frac{Wl}{4M}E=\frac{Wl^3}{16Mh}E$.

56. The web may be neglected. The difference between the result and the experiment is probably due to the tensile resistance of the metal being greater than 7 tons, owing to the smallness of the casting.

57. The whole of one flange and the web may be put as resisting the tensile strain, and $\frac{1}{8}$th their area at the extreme edge of the other flange as resisting the compressive strain, the rest of this flange being neglected. We have only taken 4 of the 7 experiments, as in the other 3 the metal must have been of exceptional quality, and even in these it must have had a greater tensile resistance than 7 tons.

66. From this point to the end the examples are all to be done by the formulæ at the end of the chapter.

67. The deflection would be too great. A span of 4 ft. should not be exceeded.

74. Two such timbers carried a heavy locomotive over this span.

78. Load 2 cwt. per square foot.

84. Take $c=2$ for the red wood. By direct experiment the value of E was found to be ·00125 for the pitch-pine and ·0025 for the red wood. The pressure applied was one ton per sq. in. which crushed the latter, but the former required two tons to crush it. The pieces tested were 50 ins. long and 10 ins. diam.

CHAPTER V.

3. The line of thrust is very nearly on the limit of the middle third at the centre of the span, and may be taken as being on it.

4. In No. 2 the line of thrust passes outside the middle third. Two bridges were built of these dimensions, and the No. 2 failed when the centres were slackened, the crown of the arch rising. The weight was, however, taken off the high side, and the centres forced upwards again by jacks. Spandril arches were then built on the high side, and the arch has since stood safely.

7 and 8. An exact agreement with the answer in these cases is hardly to be expected.

9. It will be found that $\dfrac{\Sigma_b^a xy}{\Sigma_b^a y^2}=3{\cdot}9.$

11. In (a) the line of thrust diverges 1 ft. $1\frac{1}{2}$ ins. from the curve of the arch, and in (b) 9 ins.

12. Here we have $\dfrac{\Sigma_b^a xy}{\Sigma_b^a y^2}=3{\cdot}9.$

13. We have $H=35\times\dfrac{{\cdot}0005}{{\cdot}0001}\times\dfrac{2\times4\left(\dfrac{10\frac{1}{2}}{12}\right)^2}{5\times185}=1{\cdot}15$

15. Let ab, cd be two diameters of the ring at right angles

to each other, and let the strain act in the line of ab. First consider this as hinged at c and d, and let M be its moment of resistance. Then we have $\frac{P}{2}\frac{cd}{2}=sM$. Again consider it as hinged at a and b, and we have $\frac{P}{2}\frac{cd}{2}=sM$ once more. Hence we may consider half P sustained by the girder cad, and half by the cantilevers ac, ad. Hence $\frac{P}{4}\cdot\frac{cd}{2}=sM$. In addition to the bending moments at c and d, a direct tensile strain has to be sustained.

16. This is a common railway bridge. The pressure in (a) can be found in the following way:—Let p be the minimum strain per square inch, and $p'+p$ the maximum. Then the total pressure $P=p\ (22\frac{1}{2}\times 12)+p'\ \frac{22\frac{1}{2}\times 12}{2}$,

and $$P.\ 12\tfrac{3}{8}=p\ (22\tfrac{1}{2}\times 12)11\tfrac{1}{4}+p'\ \frac{22\frac{1}{2}\times 12}{2}15.$$

From the diagram of strains, P will be found to be about 105 cwt. In (b), where the divergence is greatest, the pressure is 70 cwt.

CHAPTER VI.

2. These must have gained some support at the ends.

10. We have $r=\sqrt{\frac{8\pi\frac{3}{8}(\frac{8}{3})^2+4(1\frac{1}{2}-\frac{3}{4})(3\frac{1}{2})^2}{8\pi\frac{3}{8}+4(1\frac{1}{2}-\frac{3}{4})\times 1\text{ in.}}}=3$ (nearly), the $\frac{3}{4}$ in. being taken off for the rivets.

11. We have $r=3$ ins. one way, and a trifle more the other.

12. We have $r=4\frac{1}{4}$ nearly.

13. For common sections, r is nearly the same in each case.

14. Here $r=\cdot 64$.

15. Here approximately $r^2=\frac{\frac{1}{2}\times\cdot 7\times\frac{3}{2}\times 2}{2\cdot 64}\cdot\frac{3}{2}$ and so $r=\cdot 77$. 3 ins. is the width of the flanges.

16. $r=(a)\ \cdot 5$, (b) $\cdot 64$ (sideways), (c) $\cdot 58$, (d) $\cdot 50$. M being taken as $\cdot 2\times 1\cdot 2$ only, for the cross strain is not central.

In the experiments there was some doubt as to the form of the ends. If they are considered as partly fixed, the difference between the theory and experiment will be considerably reduced.

23. Take 24 tons as the load per pile instead of 15 which would be the actual load.

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